Excitons and Polaritons in Novel Two-dimensional Materials, Embedded in a Microcavity

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- Head of the **Laboratory of Nanostructures at the Institute of Spectroscopy**, Russian Academy of Sciences
- Professor at **Moscow Institute of Physics and Technology** and **National Research University Higher School of Economics**
- Influenced the fields of **nanostructures, low-dimensional semiconductor systems, quantum optics, and nanophotonics**
- Proposed **Bose–Einstein condensation of dipolar (indirect) excitons**
- Developed groundbreaking theories on **superfluid states in double-layer electron-hole systems**
- Explored **Josephson-type effects, drag effects, and anomalous magnetic field behaviors**
- Predicted the **Dynamical Lamb Effect** in cavity quantum electrodynamics
- Over **700 peer-reviewed papers**, including **20 reviews and monographs**
- Contributions across **excitonics, plasmonics, nano-optics, photonic crystals, nanotechnology, cavity QED, graphene, novel 2D materials. and topological insulators**
- Supervised **over 40 PhD students** and remained deeply involved in their careers
- Fellow of the **European Academy of Sciences**

Yurii Lozovik
(1 Nov 1937 – 24 June 2024)

Read full obituary at ***Physics Today*** or scan QR Code to the right:

[1] O. Berman, G. Gumbs, R. Kezerashvili, I. Lerner, N. Voronova, V. Yudson, and K. Ziegler, Yurii Lozovik, Phys. Today **2024**, 43765 (2024).

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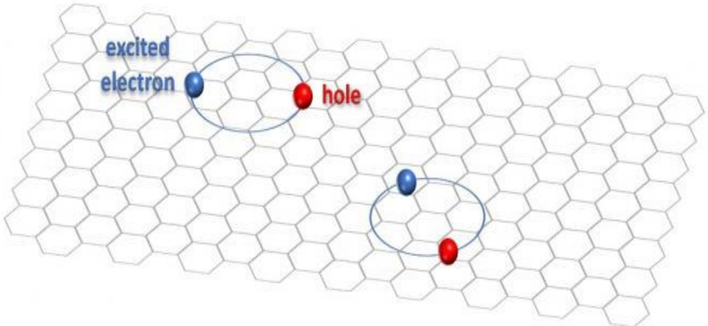
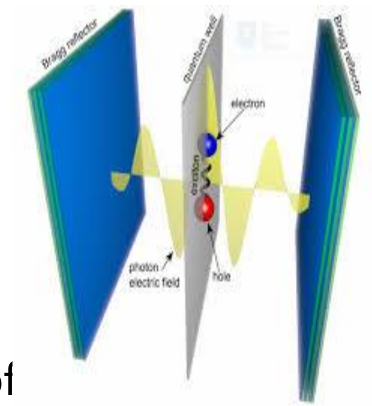
Excitons and Polaritons



- Excitons are quasiparticles formed by the bound state of an electron in the conduction band and a hole in the valence band

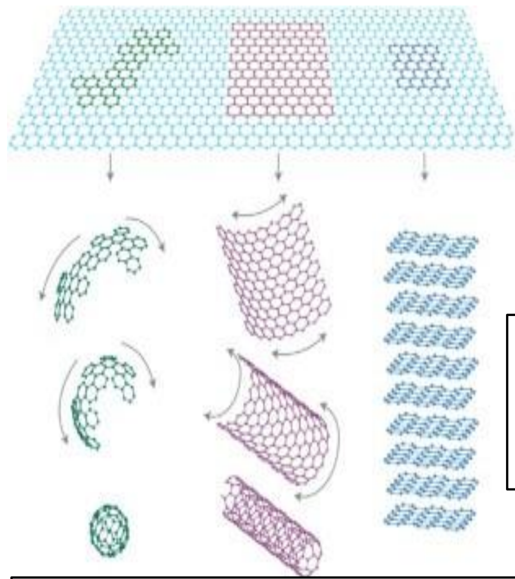
- Can usually be created by the absorption of a photon, and can decay by emitting a photon

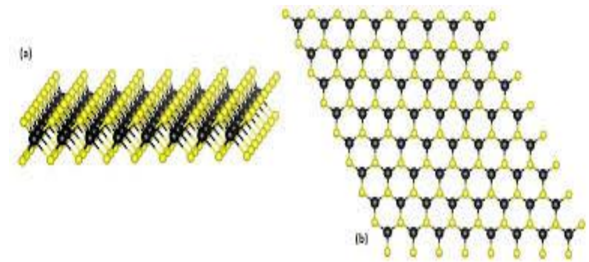
- When excitons are confined in an optical microcavity, a stable supperposed state of excitons and cavity photons called exciton-polariton can appear



Two-dimensional Materials

- Crystalline structures consisting of only few layers of atoms





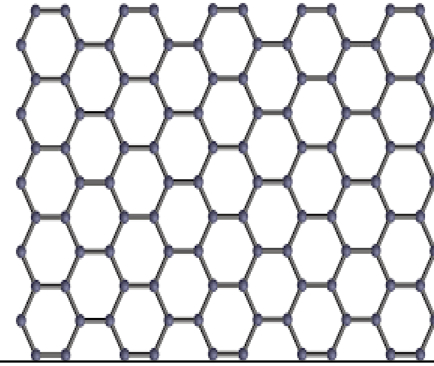
Transition metal dichalcogenides (TMDCs) –
C. Jin *et al. Nat. Nano.* **13**, 994–1003, 2018

Graphene – A. Geim, and K. Novoselov, *Nat. Mat.* **6** 183-191, 2007

- Possible applications of nanomaterials range from medicine, to civil engineering, to electronics, and much more

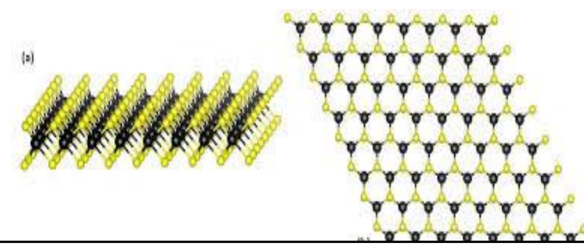
Two-dimensional Materials

Graphene monolayer



A. Geim and K. Novoselov were awarded the 2010 Nobel Prize in Physics

Graphene: a crystalline structure consisting of a single layer of graphite.



Transition metal dichalcogenides (TMDCs) –
C. Jin *et al. Nat. Nano.* **13**, 994–1003 (2018)

Transition metal dichalcogenide (TMDC): Crystalline structure of the form MX2 , Where M is a transition metal (W, Mo, etc.), and X is a chalcogen (S, Se or Te). Atoms are arranged in a honeycomb lattice, with auternating sites of one metal atom (black) in a central position and two chalcogen atoms (yellow) on an upper and lower sublayers.

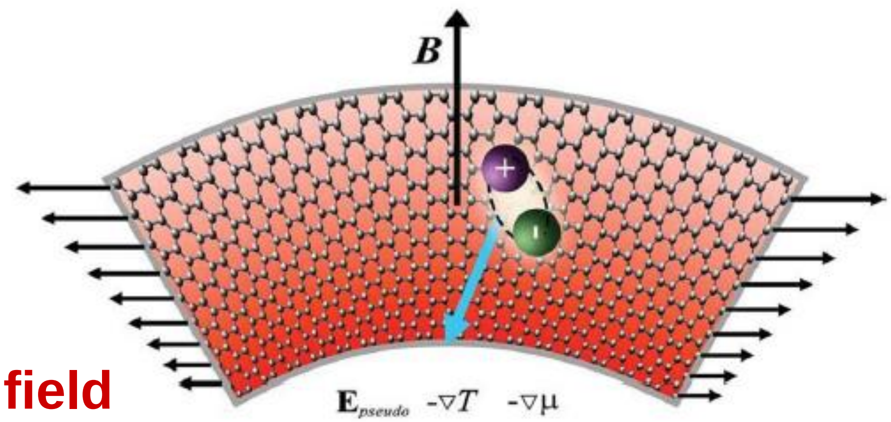
scientific reports

O. L. Berman, R. Y. Kezerashvili, Y. E. Lozovik, and K. Ziegler, *Sci Rep* 12, 2950 (2022).

OPEN

Strain-induced quantum Hall phenomena of excitons in graphene

Excitons are quasiparticles formed by the bound state of an electron (-) from the conduction band and a hole (+) from the valence band in semiconductors



Strain generates pseudomagnetic field

Quantum Hall effect for excitons

Zero magnetic field quantum Hall effect

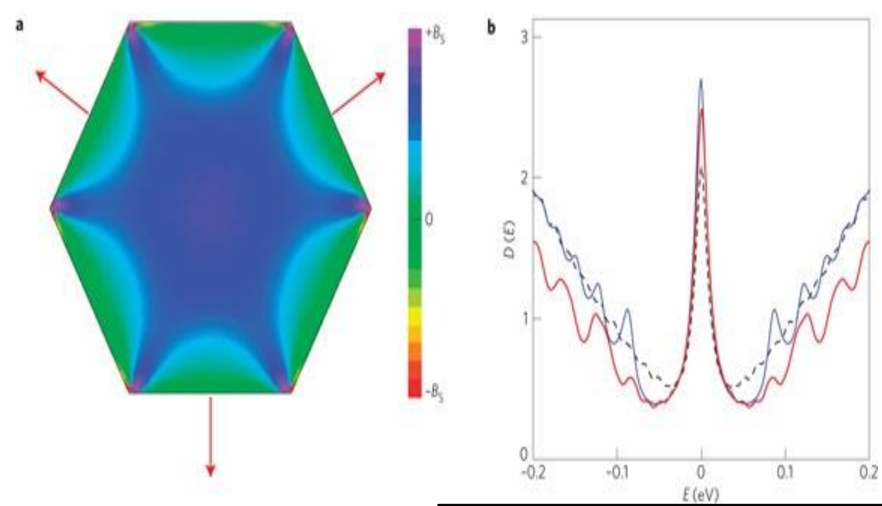
In intra-valley excitons (with an electron and a hole in the same valley), opposite pseudomagnetic fields act on electrons and holes.

In intervalley excitons (with an electron and a hole in different valleys), the same pseudomagnetic field acts on electrons and holes.

Strain-Induced Pseudomagnetic Field

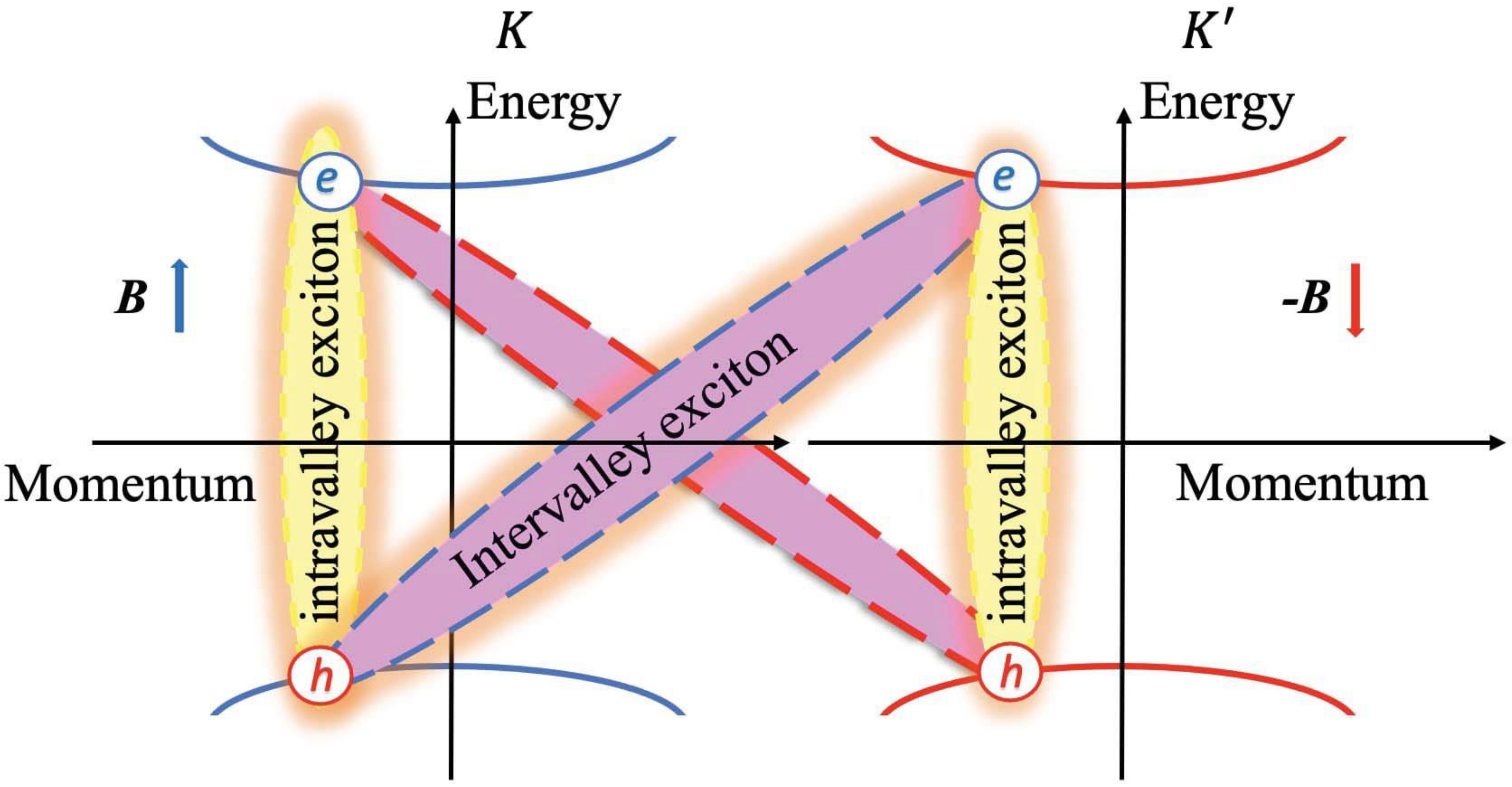
Effect of Strain in electrons and holes

- Electrons and holes in strained sheets of graphene behave as if they are subject to a magnetic field – Pseudomagnetic field (PMF)
- Effect of the PMF is charge independent – electrons and holes react the same way
- Intensity of the PMF is proportional to the deformation caused by the strain



F. Guinea, M. Katsnelson, and A. Geim *Nature Phys* **6**, 30–33 (2010).

The schematic band structure and electronic dispersions in the graphene monolayer for bright intravalley and dark intervalley pseudo magnetoexcitons in the K and K' valleys. The second intervalley PME is shown by the dashed curve.

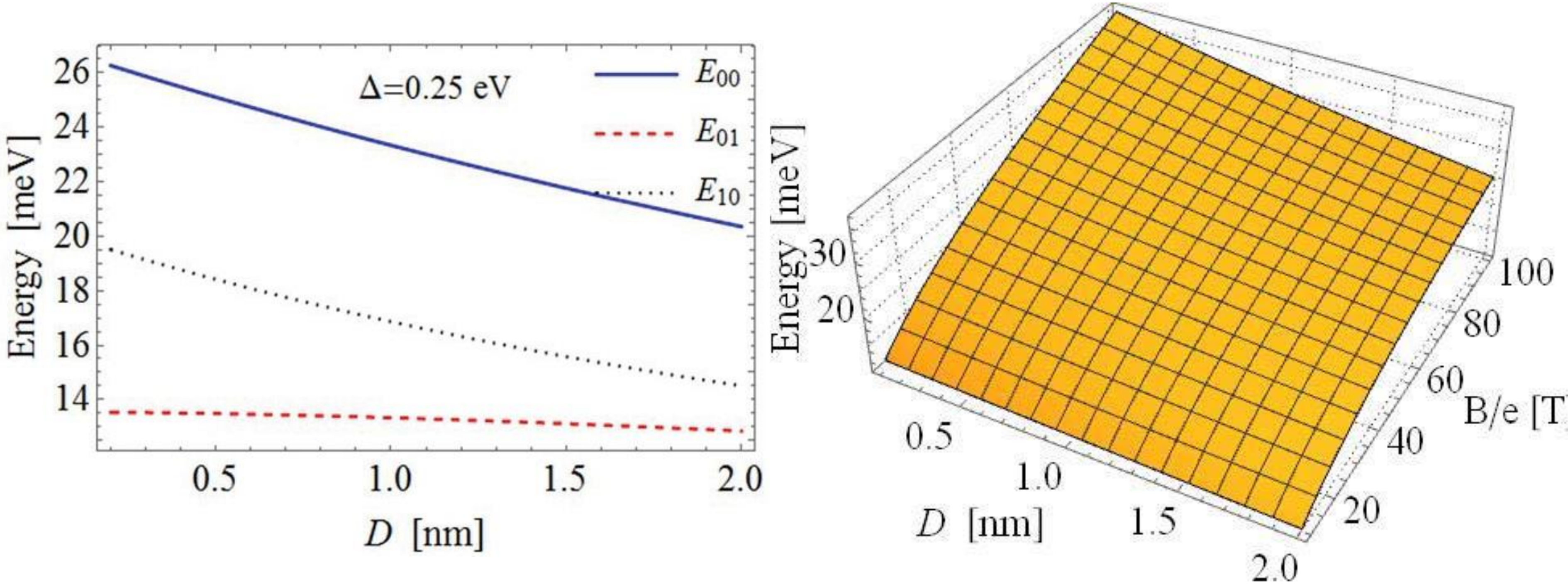


Dirac equation for a pair of an electron and a hole

$$(H_0 + V(|\mathbf{r}_1 - \mathbf{r}_2|))\Psi = \mathcal{E}\Psi$$

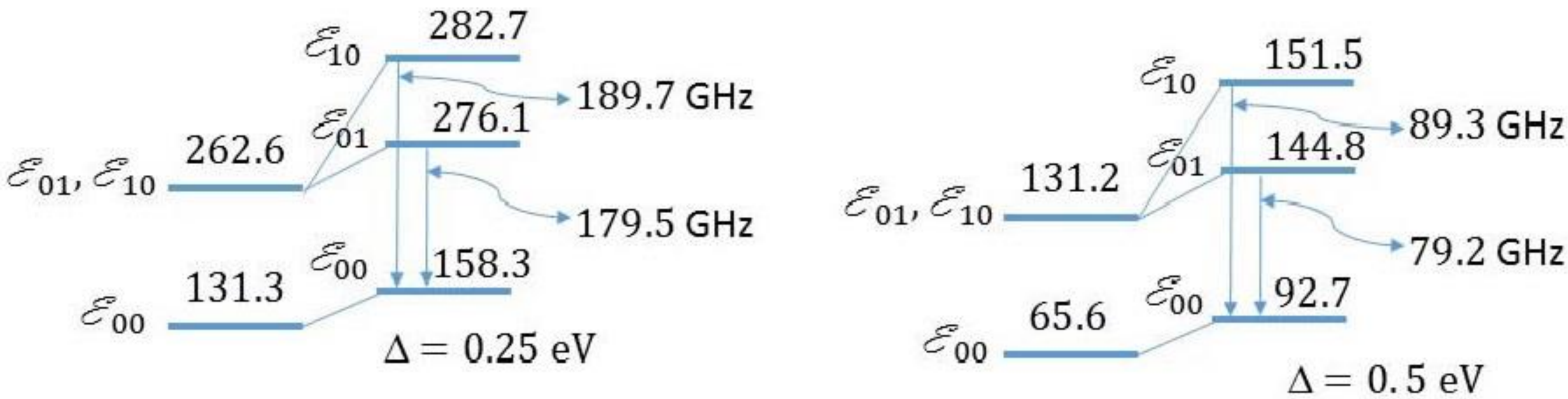
$$H_0 = v_F \sum_{j=1}^{2} \begin{pmatrix} 2\Delta/v_F & i\hbar\partial_{x_j} + A_x(\mathbf{r}_j) + \hbar\partial_{y_j} - iA_y(\mathbf{r}_j) \\ i\hbar\partial_{x_j} + A_x(\mathbf{r}_j) - \hbar\partial_{y_j} + iA_y(\mathbf{r}_j) & -2\Delta/v_F \end{pmatrix}, \quad \Psi = \begin{pmatrix} \psi_1(\mathbf{r}_1, \mathbf{r}_2) \\ \psi_2(\mathbf{r}_1, \mathbf{r}_2) \end{pmatrix}$$

In stark contrast to the vector potential of the electromagnetic field, the strain induced effective vector potentials A(r1) and A(r2), acting on an electron and a hole, forming PME, are not coupled to the charges of the particles and have the same sign in the Hamiltonian, and these potentials act on e and h the same way, and the quasi-Lorentz force due to PMF can be exerted on a moving PME.



Left panel: the dependence of energies $E'_{n,\tilde{n}}$ of indirect PMEs on the separation D between gapped graphene layers. Calculations performed for the value of magnetic length l that corresponds to $B/e = 50$ T. Right panel: the dependence of the energies of indirect PMEs $E'_{\tilde{n},\tilde{m}}$ on the separation D between gapped graphene layers and PMF B/e.

Schematic diagrams for the energy levels and transitions of the direct PME in the strained monolayer graphene. Calculations performed using RK potential for the value of magnetic length l that corresponds to $B/e = 50$ T and for $\Delta = 0.25$ eV and $\Delta = 0.5$ eV. The energy levels are given in meV. The diagrams are not drawn to scale.



(1)The stability of the integer quantum Hall effect for PMEs is determined by the energy gap between LLs, which is proportional to the PMF.

(2) Landau level quantization for PMEs can be revealed by optical spectroscopy.

STRAIN-INDUCED INTEGER QUANTUM HALL (IQHE) PHENOMENA FOR PMES

(1)When graphene is under strain, shear strain induces a **PMF**, while the dilatation gives rise to an effective scalar potential which results in the pseudoelectric field, acting on the charge carriers independently of the sign of charge, contrary to ordinary electric field.

(2) The pseudoelectric field Epseudo can be chosen to be normal to the PMF.

(3) The other option is to use laser illumination on the edge of samples, which creates
gradients of the temperature T and/or chemical potential. This drives electrons (e) and holes (h) in the same direction. The latter can trigger the flow of PMEs without breaking the bound states of e and h.

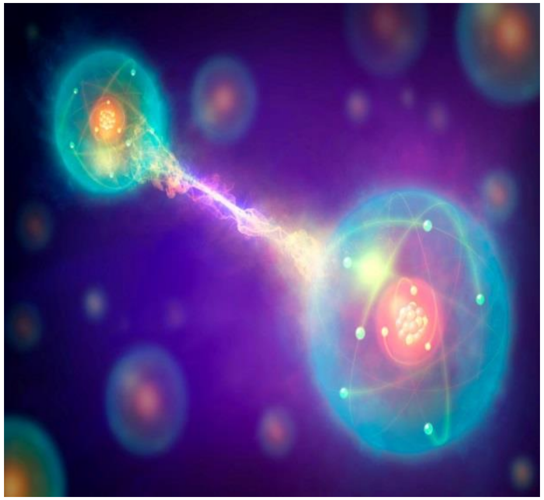
STRAIN-INDUCED FRACTIONAL QUANTUM HALL (FQHE) PHENOMENA FOR PMES

(1) The degeneracy d of the LLs n is given by $d = 2BS/\Phi_0$, where $\Phi_0 = h/2$ is the quantum of pseudomagnetic flux and the factor 2 appears due to the same action of the PMF an electron and a hole.
One can control the fillng factor of the LLs $v = N/d$ (N is the number of the PMEs) either by changing the strain, inducing the PMF, or by laser pumping changing the number N of the PMEs.

(2) One can observe not only the IQHE but also the FQHE for PMEs. For example, to observe the FQHE at the filling factor $v = 1/3$, it occurs that one needs the PMF B corresponding to four (but not three as for the 2DEG in a magnetic field) quanta of pseudomagnetic flux accounting for one bosonic PME. In this case, a composite fermion can be formed via attachment of one pseudomagnetic flux quantum to one PME, and these composite fermions with three remaining pseudomagnetic flux quanta form the FQHE state at $v = 1/3$.

Quantum Entanglement

- A degree of bonding that has no classical counterpart



- Measures performed on one part of an entangled pair immediately modify the state of the other part

A. Einstein, B. Podolski, and N. Rosen, *Phys. Rev.* **47** 777 (1935)
N. Bohr, *Phys. Rev.* **48** 696 (1935).

- Quantified by entanglement monotones, such as Concurrence, Negativity, Mutual Information, three-π, and others

PHYSICAL REVIEW B **106**, 104304 (2022)

Quantum entanglement between excitons in two-dimensional materials

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[2] *The Graduate School and University Center of The City University of New York, 365 5th Avenue, New York, New York 10016, USA*
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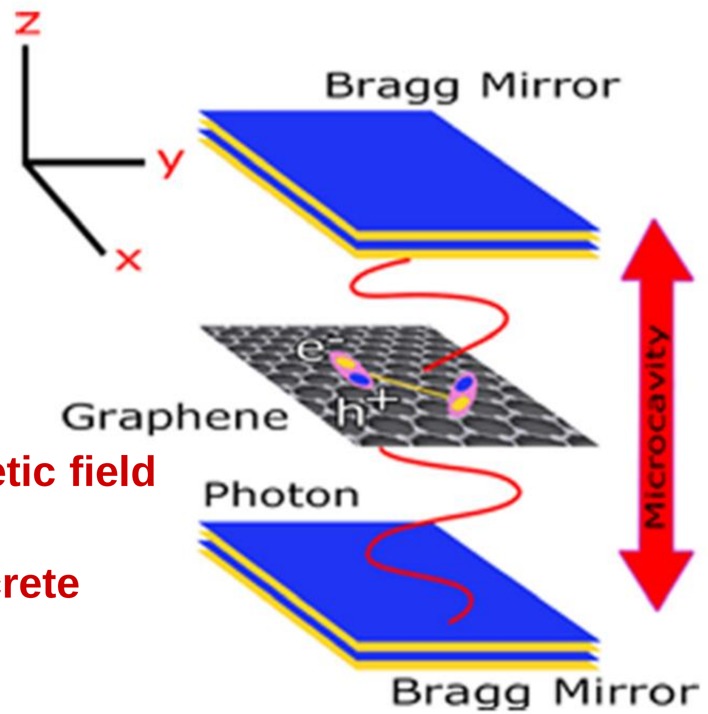
[4] *Institute of Spectroscopy, Russian Academy of Sciences, Troitsk, Moscow 142190, Russia*
[5] *Research University Higher School of Economics, Moscow 101000, Russia*



Strain generates pseudomagnetic field

The exciton spectrum is descrete

Excitons in PMF

Strain-Induced Pseudomagnetic Field

- Excitons in strain-induced PMF have a discrete, dispertionless energy set

$$\varepsilon_{n,\tilde{n},\tilde{m}} = \varepsilon_{0n\tilde{n}} + E'_{\tilde{n},\tilde{m}}$$

$$\varepsilon_{0n,\tilde{n}} = \Delta + \hbar\omega_c(1 + n + \tilde{n})$$

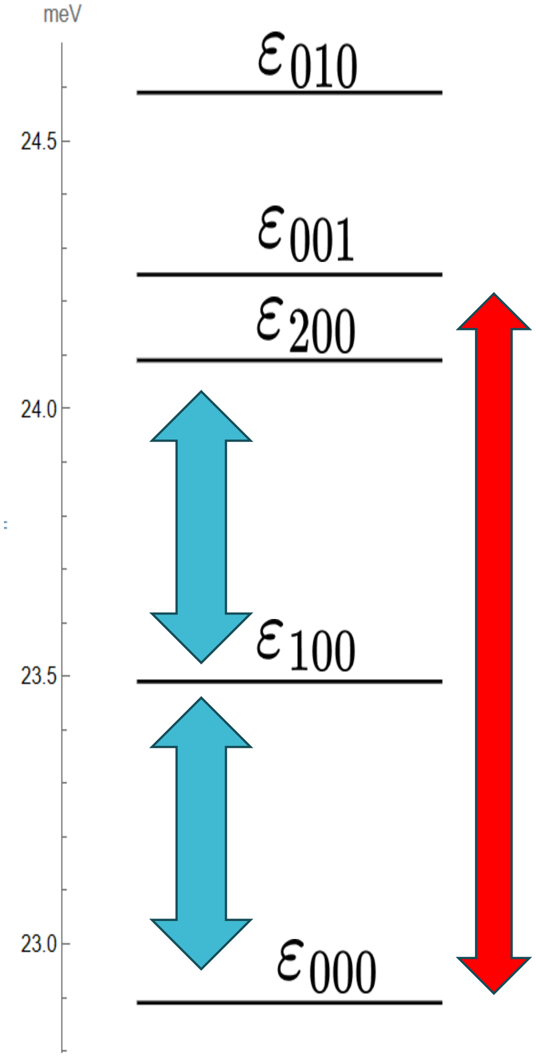
$$E'_{0,0} = -E_0 \qquad E'_{0,1} = -\frac{E_0}{2} \qquad E'_{1,0} = -\frac{3E_0}{4}$$

$$E_0 = \sqrt{\frac{\pi}{2}}\frac{ke^2}{\varepsilon_d l} \qquad l = \sqrt{\frac{\hbar}{B}}$$

O. L. Berman, R. Y. Kezerashvili, Y. E. Lozovik, and K. Ziegler, *Sci Rep* 12, 2950 (2022).

Strain-Induced Pseudomagnetic Field

Excitons in PMF



The energy gap between subsequent states with the same value of quantum numbers ñ and m̃ is always the same. The energy gap between states with different values of ñ and m̃, however, is different for each set of quantum numbers.

Tavis-Cummings Hamiltonian

- If N excitons Interact with a single cavity mode whose photons are in or near resonance with the energy gap between the exciton ground state and state 001 , excitons can be effectively treated as qubits

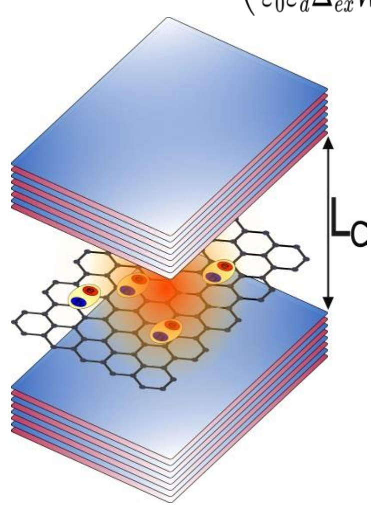
M. Tavis, and F. Cummings, *Phys. Rev.* **170**(2) 379 (1968).

- The dynamics of the composite system in the rotating wave approximation will be governed by the Tavis-Cummings Hamiltonian:

$$\hat{H}_0 = \hbar\omega_c \hat{a}^\dagger \hat{a} + \sum_{j=1}^{N} \left(\frac{\Delta_{ex}}{2} \sigma_{zj} + g(\hat{a}\sigma_{+j} + \hat{a}^\dagger \sigma_{-j}) \right)$$

$$g = e v_F \left(\frac{\pi \hbar^2}{\varepsilon_0 \varepsilon_d \Delta_{ex} W} \right)^{\frac{1}{2}}$$

$a^\dagger$: creation operator for cavity photons
g : Rabbi coupling
ω_c : frequency of cavity photons
Δ_{ex} : exciton energy gap
σ_{zj} : Pauli matrix for the $j-$th exciton
$\sigma_{\pm j}$: Creation and annihilation operators for the $j-$th exciton
v_F : Fermi velocity for electrons in graphene
W : volume of microcavity



Master Equation

- If the cavity is coupled to a coherent light source in resonance with it's mode, the hamiltonian becomes

$$\hat{H} = \hat{H}_0 + \hbar R_P \left(e^{+i\omega_L t}\hat{a} + e^{-i\omega_L t}\hat{a}^\dagger\right)$$

- When we take into consideration leaky cavities, the system will evolve following the master equation

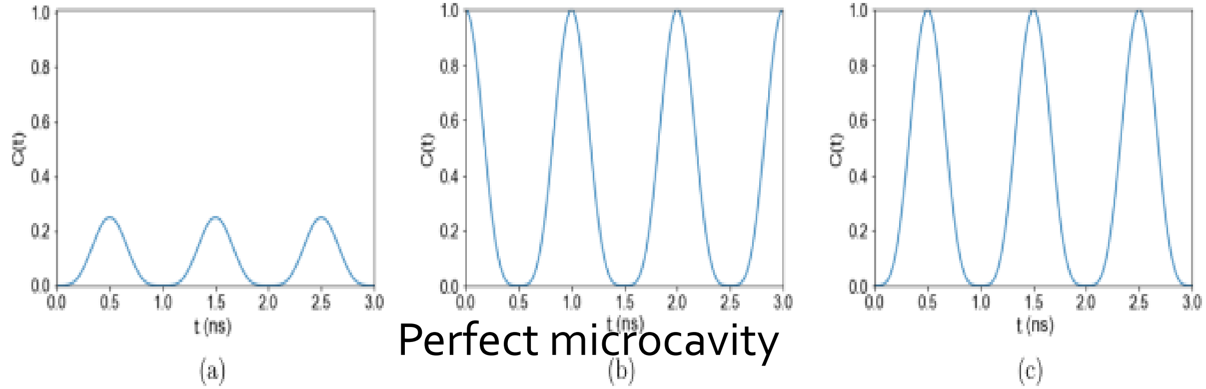
$$\hbar\dot{\rho} = \frac{1}{i}\left[\hat{H}, \rho\right] + \gamma_c \mathcal{L}(\hat{a})\rho$$

$$\mathcal{L}(\hat{O})\rho = \hat{O}\rho\hat{O}^\dagger - \frac{1}{2}\left\{\hat{O}^\dagger\hat{O}, \rho\right\}$$

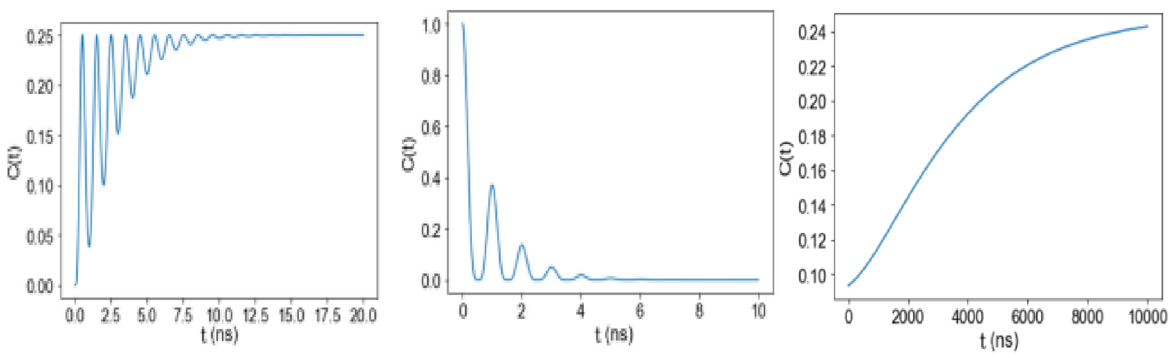
R_P : pumping rate
ω_L : frequency of laser photons
ρ : composite system's density matrix
γ_c : cavity decay rate

Numerical Results

2 excitons in strained graphene without an external pumping source: concurrence



Perfect microcavity

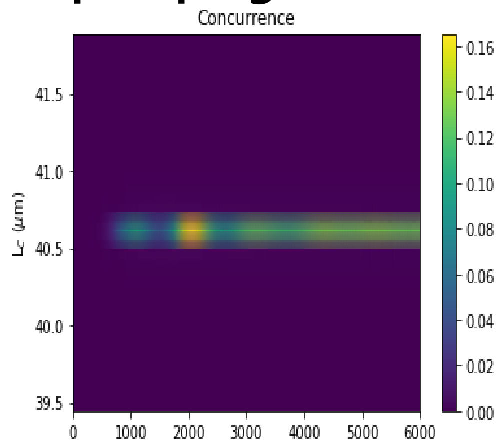


Leaky microcavity

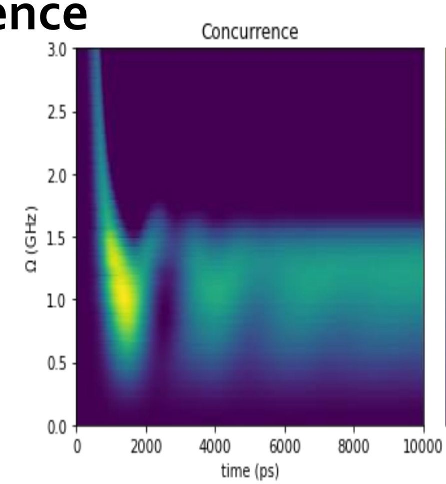
G. P. Martins, O. Berman, G. Gumbs, and Y. Lozovik - *Phys. Rev. B.* **106**, 104304 (2022).

Numerical Results

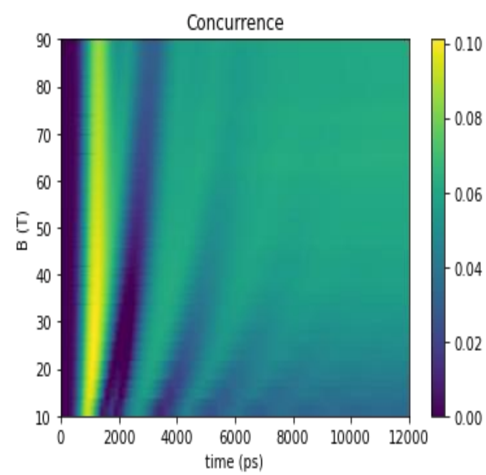
2 excitons in strained graphene with an external pumping source: concurrence



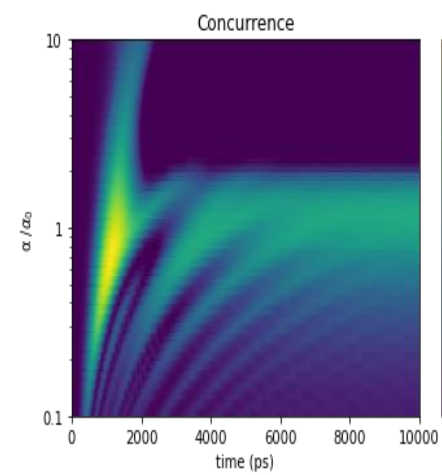
Microcavity length
(resonance happens at $L_C \approx 40.6\ \mu$m)



Laser pumping rate
(cavity decay rate $\gamma_c \approx 1.5$ GHz)



Intensity of PMF



Cavity volume and dielectric constant ($\alpha = W \times \varepsilon_d$)

G. Martins, O. Berman, G. Gumbs, and Y. Lozovik - *Phys. Rev. B.* **106**, 104304 (2022).

Analysis

- We calculated the time evolution of the concurrence for multiple initial conditions
- We saw the existence of a maximally entangled state that is protected from decay
- A leaky microcavity can, sometimes, enhance the amount of entanglement between qubits, which agrees with the result of Plenio *et al.* (PRA 1999).

PHYSICAL REVIEW B **111**, 045425 (2025)

Long-lived quantum entanglement of multiple qubits: Excitons in strained graphene

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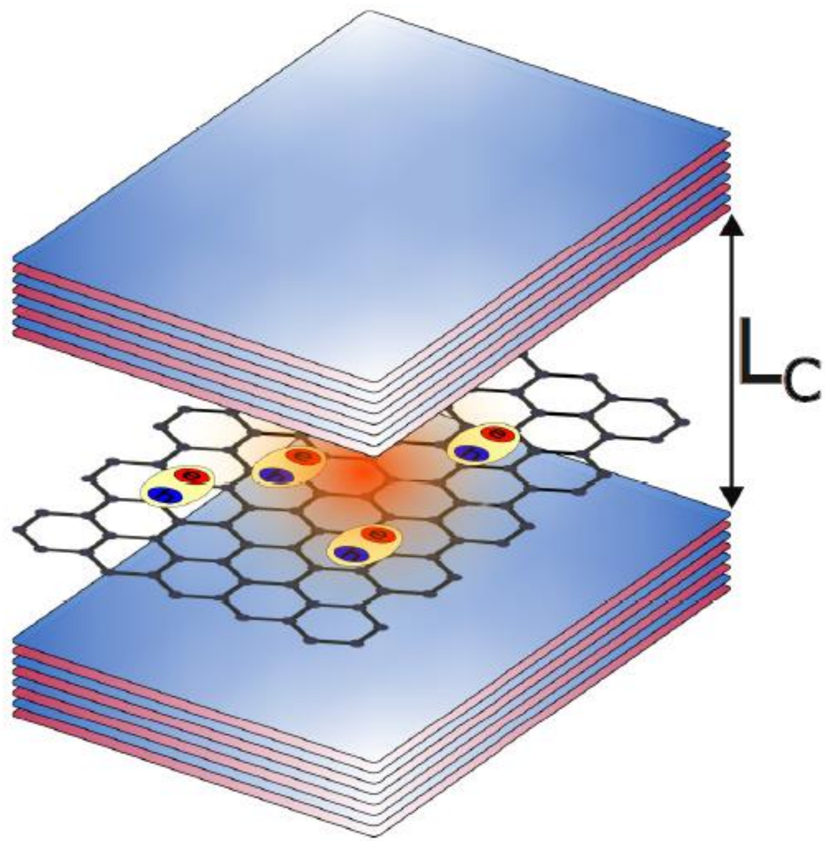
Strain generates pseudomagnetic field

The exciton spectrum is descrete



Total Negativity

- Entanglement measures usually try to quantify the amount of entanglement between one part of a multipartite system (e.g. part A) and the remainder of the system (e.g. parts B and C), but miss entanglement contained within other parts (entanglement between parts B and C)

$$|GHZ\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

- Entanglement can be shared by sets of more than 2 subsystems, like in the GHZ state
. In such a maximally entangled state, measures on only 2 qubits cannot capture any entanglement whatsoever.

- Some entanglement measures try to evaluate the amount of entanglement shared between sets of 3 qubits, such as the three-π and the three-tangle, but generalizing this for larger sets becomes increasingly complicated.

- Since entanglement is always shared, summing over the amount of entanglement experienced by each subsystem to the remaining subsystems always overcounts the amount of entanglement by a factor of at least 2.

Total Negativity

- We proposed a simple way to estimate upper and lower bounds for the overall amount of entanglement in a system of N qubits. Since entanglement is shared by at least 2 qubits, and at most all N qubits, summing over the negativities $\mathcal{N}_i$ overcounts the overall amount of entanglement by a factor of at least 2 and at most N. Namely, the total negativity $\mathcal{N}_{tot}$ must obey

$$\frac{1}{N}\sum_{1}^{N}\mathcal{N}_i \leq \mathcal{N}_{tot} \leq \frac{1}{2}\sum_{1}^{N}\mathcal{N}_i$$

$$\mathcal{N}_i \equiv \mathcal{N}_0 \ \forall i$$

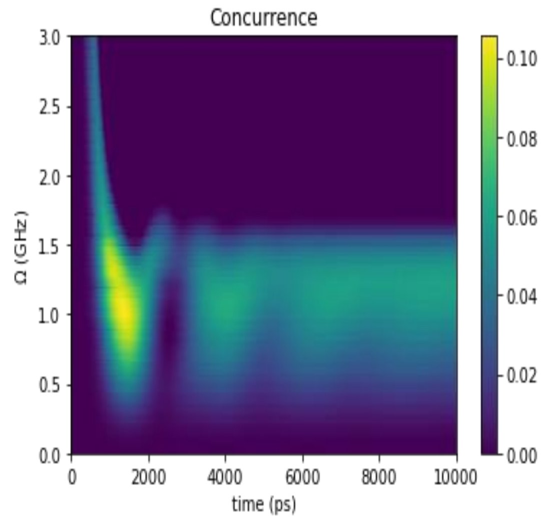
In cases in which the qubits are in a symmetric state

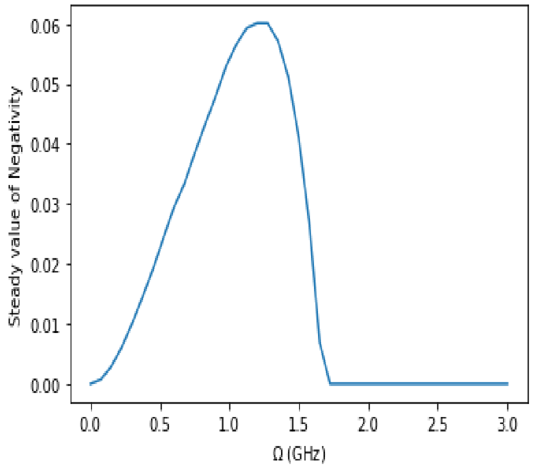
$$\mathcal{N}_0 \leq \mathcal{N}_{tot} \leq \frac{N}{2}\mathcal{N}_0$$

all considered initial states are symmetric under an exchange of qubits

Analysis

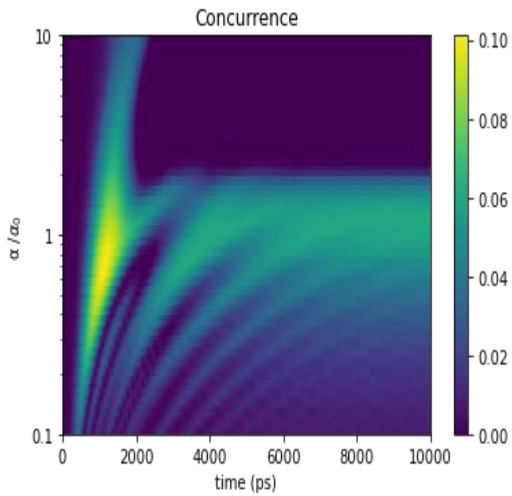
- Quantum entanglement reaches a maximum when the laser pump rate is at an optimal value close to the cavity decay rate and goes sharply to zero at higher pump rates

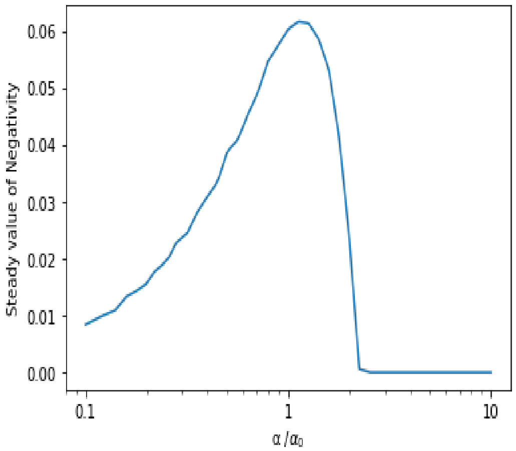




Analysis

- There is an optimal and finite value for the cavity volume (and, therefore for the Rabi coupling g) for maximal entanglement.
- Lasting entanglement goes sharply to zero for cavities larger than the optimal volume

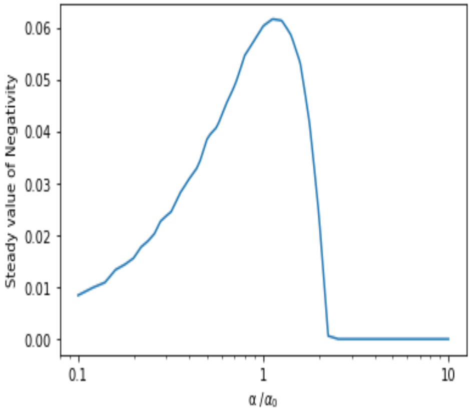




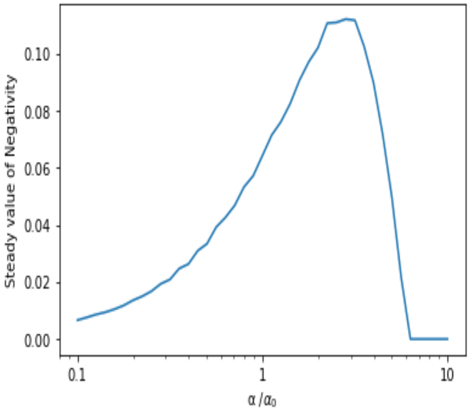
Analysis

- The same kind of pattern is observed for the dependency of lasting quantum entanglement in both the intensity of the PMF, and of the cavity volume

- Optimal value for the cavity volume is significant higher for 3 excitons when compared with two excitons



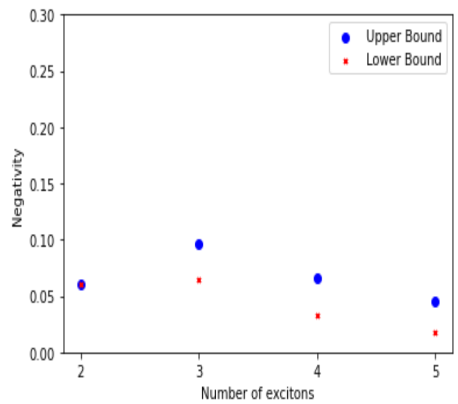
2 excitons



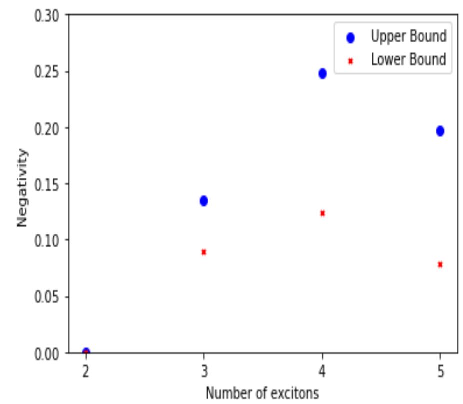
3 excitons

Analysis

- Freely increasing the number of excitons in the system does not necessarily increase the amount of quantum entanglement.
- Maximum entanglement happens at an optimal number of excitons that depend on the cavity volume (and, therefore, on the intensity of the Rabi coupling g).
- Quantum entanglement appears to depend on the excitonic density within the microcavity.



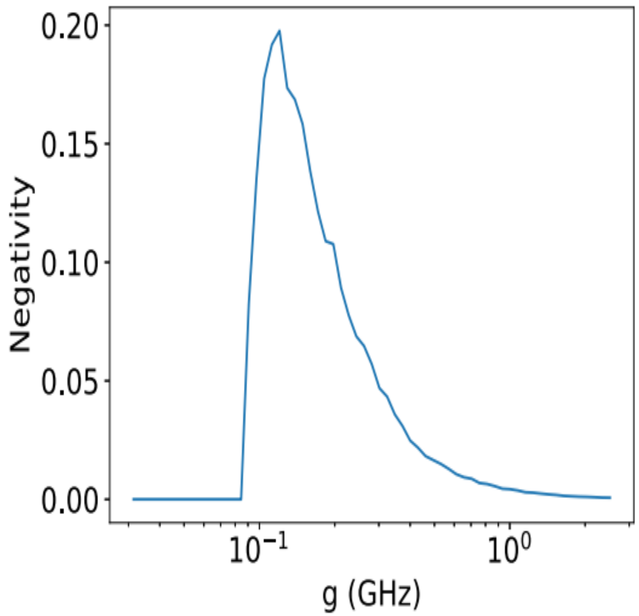
$$\frac{\alpha}{\alpha_0} = 1.0$$



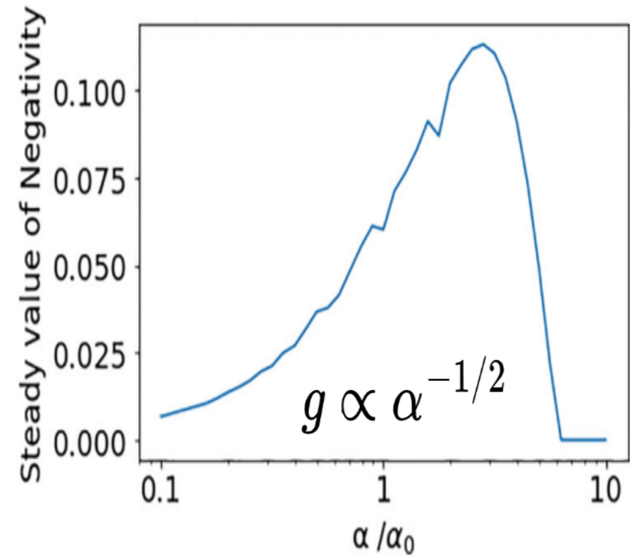
$$\frac{\alpha}{\alpha_0} = 4.0$$

Other Systems

Trapped Atoms



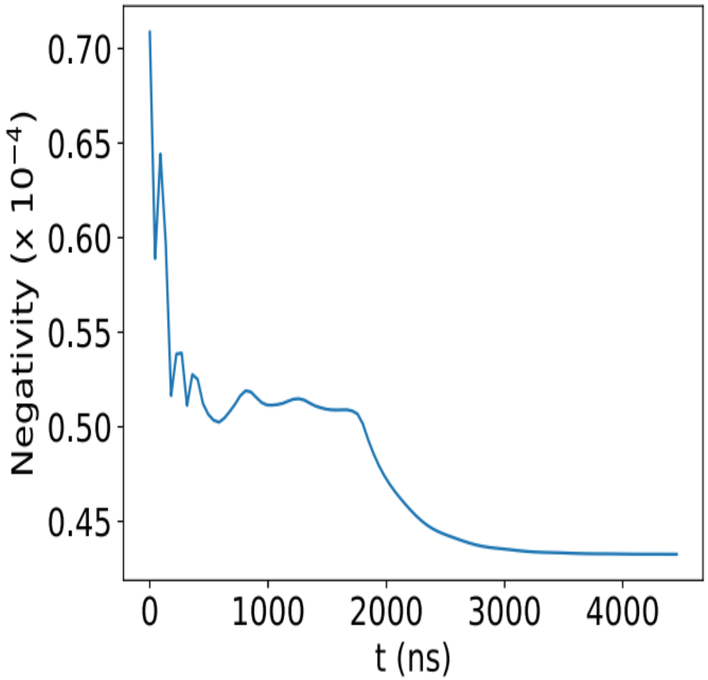
Excitons in strained graphene



- **Similarity**: Trapped atoms and excitons behave in a similar way when within a microcavity.
- **Maximum Entanglement**: Occurs when photon lifetime in the microcavity is around 1/g, with few photon-qubit interactions.
- **Multiple Interactions**: When 1/g is much larger than photon lifetime, averaging out to zero -> entanglement decreases asymptotically to zero.
- **Unlikely Interactions**: When 1/g is smaller than photon lifetime, virtually no interactions happen -> entanglement decays sharply to zero.

Other Systems

Superconducting Qubits coupled to a wave guide



In a system of superconducting qubits coupled to a waveguide such as in PRA **69**, 062320 (2004), simply pumping the system with a coherent source of photons will not create sensible entanglement. This happens because, in this system, the lifetime of waveguide photons is very big compared to the average time for qubit-photon interactions (1/g). In this case, many interactions take place for each cavity photon. The final result in terms of entanglement created averages to a negligible value.

Conclusions

- Our proposed system naturally creates entangled excitons by pumping an optical microcavity with coherent photons.
- We proposed a way of estimating the overall amount of entanglement in a multipartite system based on the negativity experienced by each individual subsystem.
- We reached the interesting result that freely adding more excitons in the composed system will usually decrease the overall amount of quantum entanglement
- Our results are easily adaptable to any system that can be approximated by the Tavis-Cummings model

G. P. Martins, O. L. Berman, G. Gumbs, and Yu. E. Lozovik - *Phys. Rev. B.* **111**, 045425 (2025)

Time Crystals

A time crystal (TC) is a quantum system of particles whose lowest-energy state is one in which the particles are in repetitive motion.

Nobel laureate Frank Wilczek (MIT), 2012:

F. Wilczek, "Quantum Time Crystals," ***Physical Review Letters*. 109, 160401 (2012).**
A. Shapere, and F. Wilczek, "Classical Time Crystals". ***Physical Review Letters*. 109,** 160402 **(2012).**

Time crystals are a time-based analogue to <u>common crystals</u> whereas the atoms in crystals are arranged periodically in space, the atoms in a time crystal are arranged periodically in both space and time.

<u>A time crystal</u> is periodic in time in the same sense that the <u>pendulum in a pendulum-driven clock</u> is periodic in time. Unlike a pendulum, a time crystal "spontaneously" self-organizes into robust periodic motion (breaking a temporal symmetry).

Time crystals may be used for quantum computer memory.

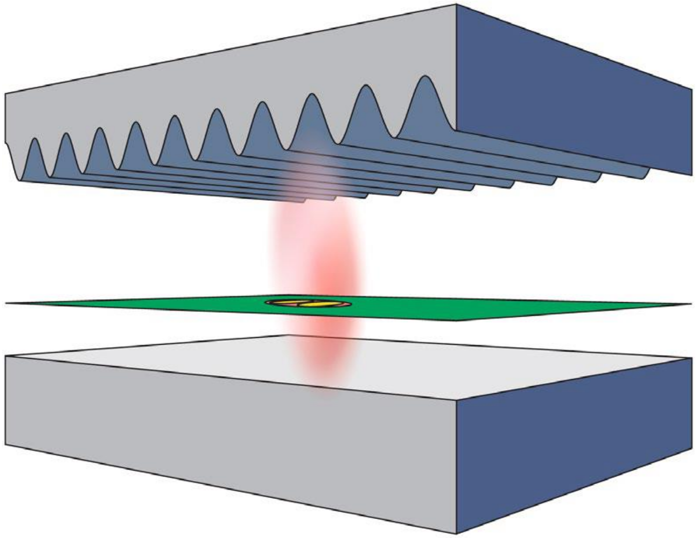
scientific reports

G. P. Martins, O. L. Berman, and G. Gumbs, *Sci. Rep.* 13, 19707 (2023).

OPEN

Polaritonic and excitonic semiclassical time crystals based on TMDC strips in an external periodic potential

Cavity polariton BEC time crystal

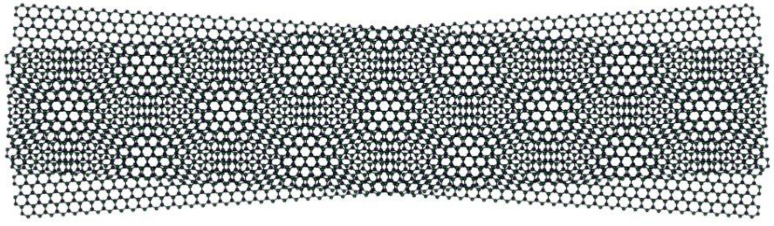


Bose-Einstein condensation: many microscopic particles behave as a single macroscopic particle.

BEC: Macroscopic number of particles occupy the lowest energy state

Polaritons are part of matter and part of light

Exciton BEC time crystal: twisted bilayer



Moiré pattern in the crystal lattice structure seen by twisting one of the layers of a bilayer TMDC.

Trapping Cavity Polaritons

D. W. Snoke, Science 298, 1368 (2002)

P. Littlewood, Science 316, 989 (2007)

D. W. Snoke and J. Keeling, Phys. Today 70, 54 (2017).

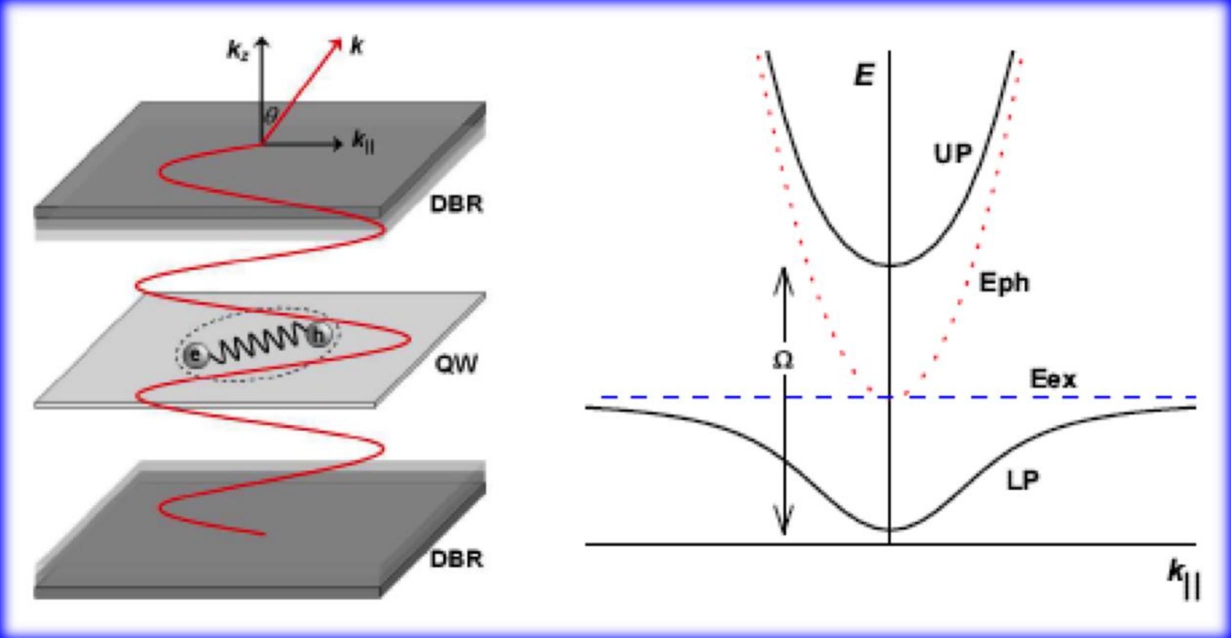
Cavity polaritons in TMDC:

Experiments:

X. Liu, et al., Nat. Photonics 9, 30 (2015).

L. C. Flatten, et al., Sci. Rep. 6, 33134 (2016).

S. Dufferwiel, et al., Nature Commun. 6, 8579 2015).



cavity photon:

$$E = \hbar c\sqrt{k_z^2 + k_{||}^2} = \hbar c\sqrt{(\pi / L)^2 + k_{||}^2}$$

quantum well exciton:

$$E = E_{gap} - \Delta_{bind} + \frac{h^2 N^2}{2m_r(2L)^2} + \frac{\hbar^2 k_{||}^2}{2m}$$

Time Crystals

A time crystal (TC) is a state of matter that shows spontaneous breaking of time translation symmetry (TTS).

F. Wilczek, Phys. Rev. Lett. 109, 160401 (2012).
A. Shapere and F. Wilczek, Phys. Rev. Lett. 109, 160402 (2012).

A mathematical criterion to verify whether a system is in a TC phase:

H. Watanabe and M. Oshikawa, Phys. Rev. Lett. 114, 251603 (2015).

For a system to be a TC, it has to show some non-trivial two-point correlator in some observable $\hat{O}$ at two different and long-apart times:

$$\lim_{|\mathbf{r}-\mathbf{r}'|\to\infty} \lim_{t-t'\to\infty} \left\langle \hat{O}(\mathbf{r},t)\hat{O}(\mathbf{r}',t') \right\rangle = c(t)$$

Here *c(t)* is some non-trivial non-stationary function.

Exciton-Polariton BEC in an Uneven Microcavity

A strip of $MoSe_2$ inside an uneven microcavity.

The cavity is composed of a plane mirror at the bottom and a spatially curved mirror on the top. The cavity width $L_C(x)$ is not constant and is a function of x

Exciton is a bound state of an electron from the conduction band and a hole from the valence band.

Exciton-Polariton is a linear superposition of an exciton and a cavity photon.

I. Carusotto and C. Ciuti, Rev. Mod. Phys. 85, 299 (2013).

Photon energy: $\varepsilon_{ph}(\mathrm{P}) = (c/n)\sqrt{P^2 + \hbar^2\pi^2 L_C{}^{-2}}$

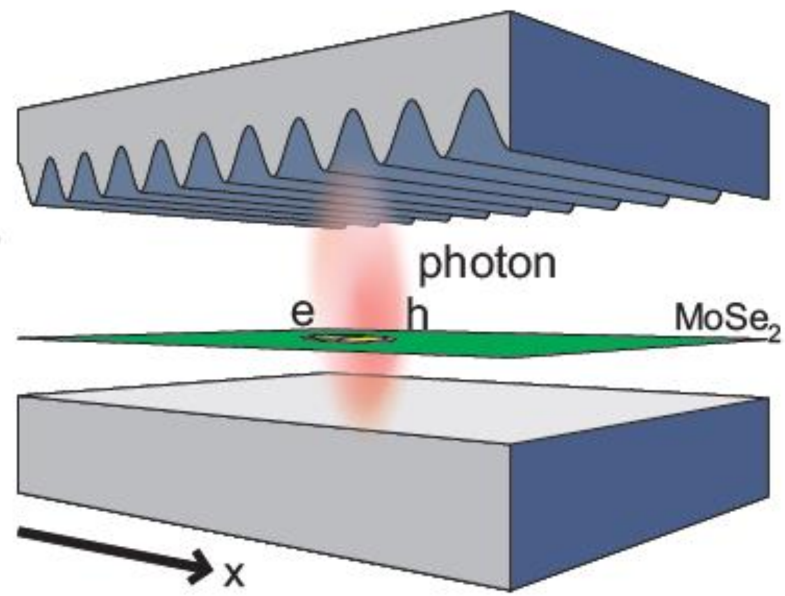
n is the index of refraction of the cavity

P is the in-plane momentum

$$L_C(x) = \frac{\hbar\pi q c}{n\left(\varepsilon_{\text{exc}} + 2V_0\cos(kx)\right)}$$

q is the number of the photon mode

ε_{exc} **is energy of creation of an exciton**



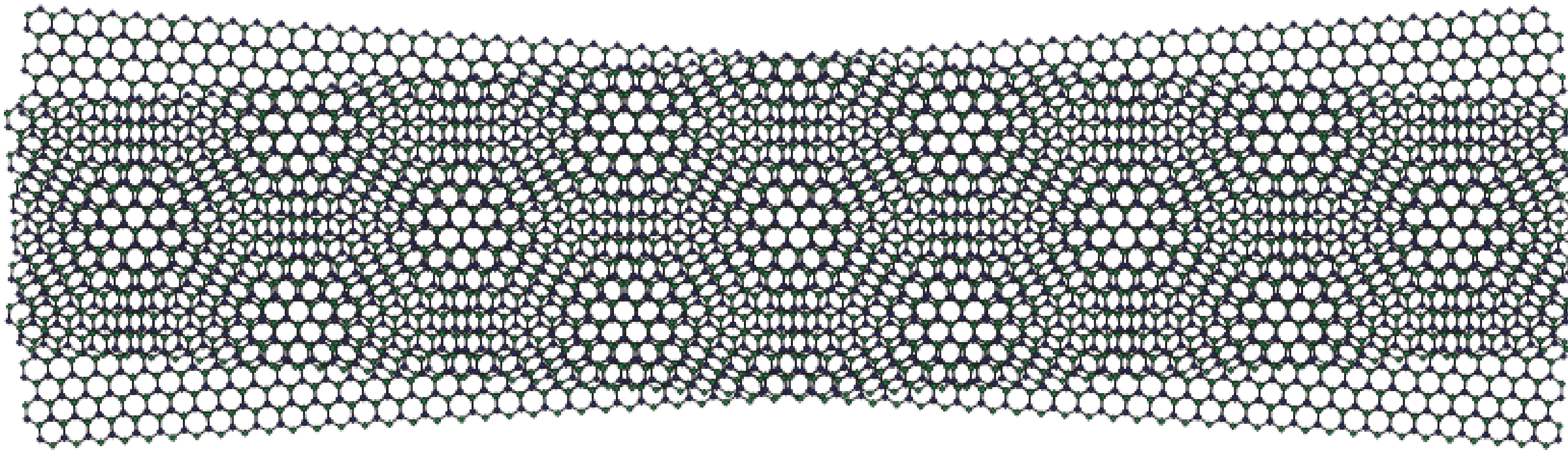
Polaritons confined in such a cavity would be subjected to an effective potential $V_{\text{eff}}(x) = V_0\cos(kx)$.

Excitonic BEC in a Twisted TMDC Bilayer

A system composed of bare excitons in a twisted TMDC bilayer.

The superposition of two layers of TMDC twisted with respect to each other will act on the excitons in it as an external periodic potential.

K. Tran et al., Nature 567, 71 (2019).



Moiré pattern in the crystal lattice structure seen by twisting one of the layers of a bilayer TMDC.

The pattern is created by the difference in atomic alignment in the upper and lower layers.

Excitons in such an environment will be subject to an external periodic potential:

$$V(\mathrm{r}) = V_0 \sum_j \cos(k\mathrm{b}_j \cdot \mathrm{r})$$
$$= 2V_0 \left(\cos(kx) + \cos k\left(\frac{x}{2} + \frac{\sqrt{3}}{2}y\right) + \cos k\left(\frac{x}{2} - \frac{\sqrt{3}}{2}y\right)\right)$$

Mathematical Framework

Mean-Field evolution:

Modified Gross-Pitaevskii equation:

J, Keeling et al, edited by N. Proukakis, D. W. Snoke, and P. B. Littlewood, Universal Themes of Bose-Einstein Condensation, (Cambridge University Press, 2017).

$$i\hbar\frac{\partial\tilde{\varphi}}{\partial t} = \left[-\frac{\hbar^2\nabla^2}{2M_P} + V_{\text{eff}}(r) + U_{\text{eff}}\left|\tilde{\varphi}\right|^2 + i\left(\gamma - \kappa - \Gamma\left|\tilde{\varphi}\right|^2\right)\right]\tilde{\varphi}.$$

$V_{eff}(r)$ is the external periodic potential ; U_{eff} is the particle-particle interaction

$\tilde{\varphi}(r)$ is the wave function of the condensate

κ, γ and Γ are the rates of single-particle loss, single-particle incoherent pumping and two-particle loss,

Beyond Mean-Field description:

In the semiclassical limit, one can find the following equation for the "classical" field ψ_C

$$i\hbar\frac{\partial\psi_C}{\partial t} = \left[-\frac{\hbar^2\nabla^2}{2M} + V_{\text{eff}}(\mathbf{r}) + U_{\text{eff}}\left|\psi_C\right|^2 + i\left(\gamma - \kappa - \Gamma|\psi_C|^2\right)\right]\psi_C + i\left(\kappa + \gamma\right)\psi_Q$$

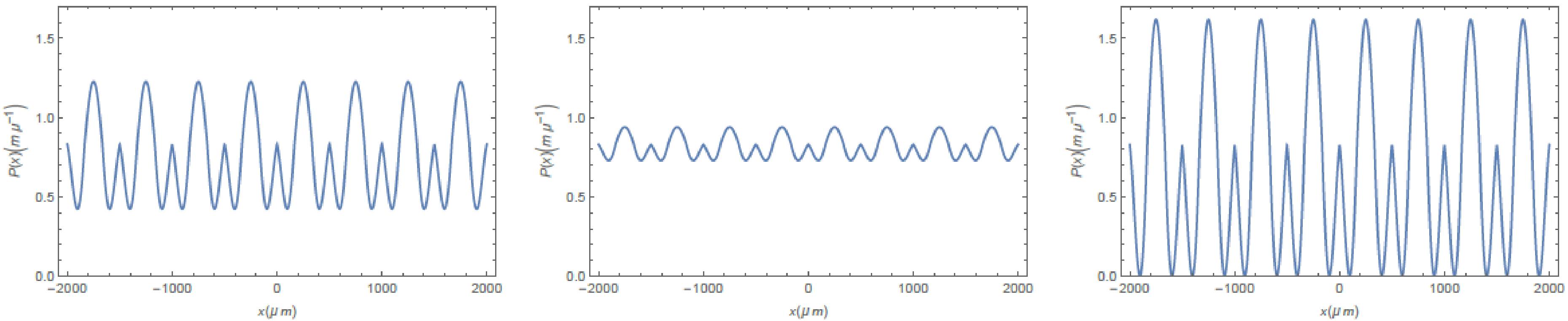
ψ_Q is the "quantum" field

the replacement $i\left(\kappa+\gamma\right)\psi_Q \rightarrow \xi(\mathbf{r},t)$, where $\xi(\mathbf{r},t)$ represents a Gaussian white noise

$$\langle\xi(\mathbf{r},t)\rangle = 0, \qquad \left\langle\xi(\mathbf{r},t)\bar{\xi}(\mathbf{r}',t')\right\rangle = \frac{(\gamma+\kappa)}{2}\delta\left(t-t'\right)\delta\left(\mathbf{r}-\mathbf{r}'\right)$$

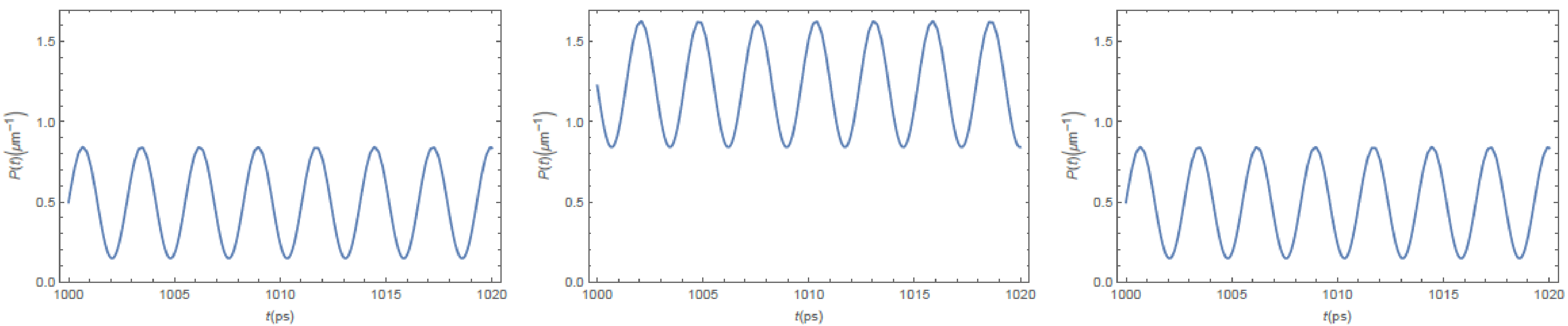
Polariton BEC density (mean-field)

we considered the length of the strip to be $L_x = 4,000$ μm, and its width to be $L_y = 1$ μm.



(a) (b) (c)

Figure 3: Polariton BEC density $P(t,x) = |\varphi(t,x)|^2$ for the polariton BEC in a ribbon of length 4000 μm at three different simulation times, at intervals of 1 ps from one another and starting at 3 ns. (a) t = 3000 ps ; (b) t = 3001 ps; and (c) t = 3002 ps.



(a) (b) (c)

Figure 4: Polariton BEC density $P(t,x) = |\varphi(t,x)|^2$ in a ribbon of length 4000 μm at three chosen positions. (a) x = $\pm$ 125.0 μm; (b) x = $\pm$ 250.0 μm; and (c) x = $\pm$ 375.0 μm.

Two-point correlation function *c(t)* (mean-field)

we considered the length of the strip to be $L_x = 4,000\ \mu\text{m}$, and its width to be $L_y = 1\ \mu\text{m}$.

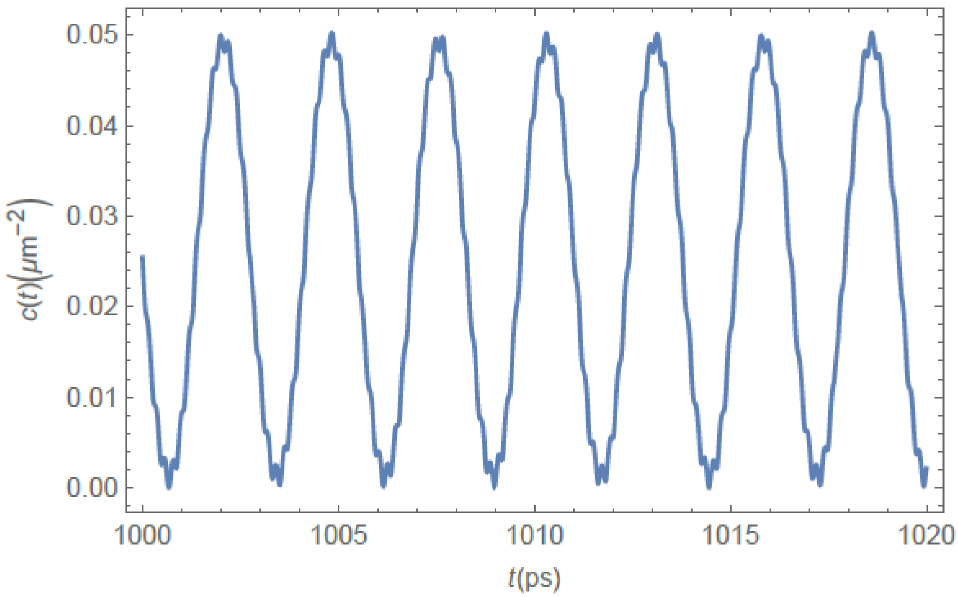


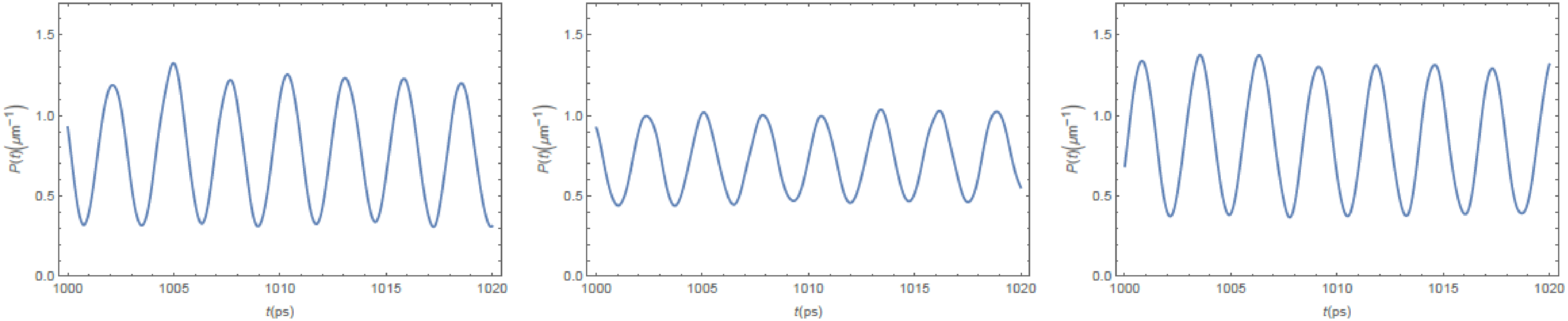
Figure 6: Two-point correlation function $c(t)$ as defined in Eq. 5 for the polariton BEC. For this plot, we considered the observable O to be the deviation between the condensate density $P(x,t)$ and the unperturbed steady-state density P_0. We took $x - x'$ to be half the length of the strip and calculated the expectation value $\left\langle \hat{O}(\mathbf{r},t)\hat{O}(\mathbf{r}',t')\right\rangle$ by taking the average value for all x while maintaining $x - x'$ constant.

$$\lim_{|\Delta\mathbf{r}|\to\infty}\ \lim_{|t-t'|\to\infty} \frac{1}{V}\int \rho(\mathbf{r},t)\rho(\mathbf{r}+\Delta\mathbf{r},t')d\mathbf{r} = c(t), \qquad P_0 = |\varphi_0|^2, \rho(x,t) = \frac{P(x,t)-P_0}{P_0}.$$

unperturbed density **the order parameter**

Polariton BEC Density and Correlation Function (beyond mean-field)

we considered the length of the strip to be $L_x = 4,000\ \mu$m, and its width to be $L_y = 1\ \mu$m.



(a) (b) (c)

Figure 7: Polariton BEC density $P(t,x) = |\varphi(x,t)|^2$ in a strip 4000 μm long at three different positions as a function of time, when quantum uncertainty is taken into consideration. (a) x = 125 μm; (b) x = 250 μm; and (c) x = 375 μm.

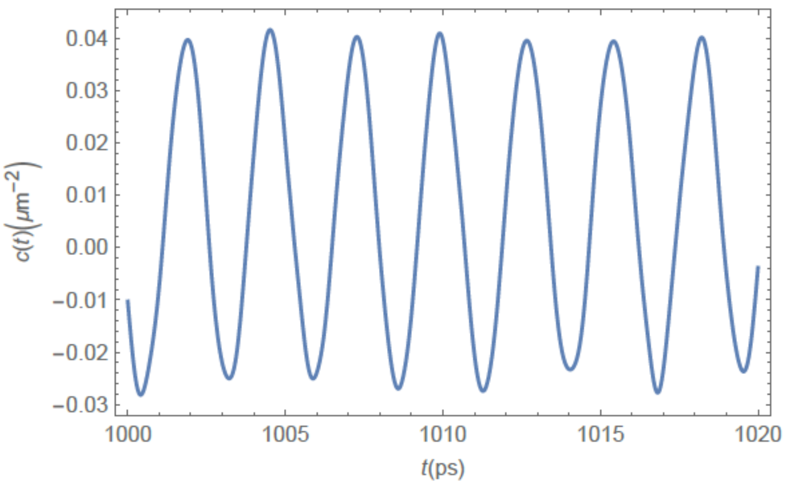
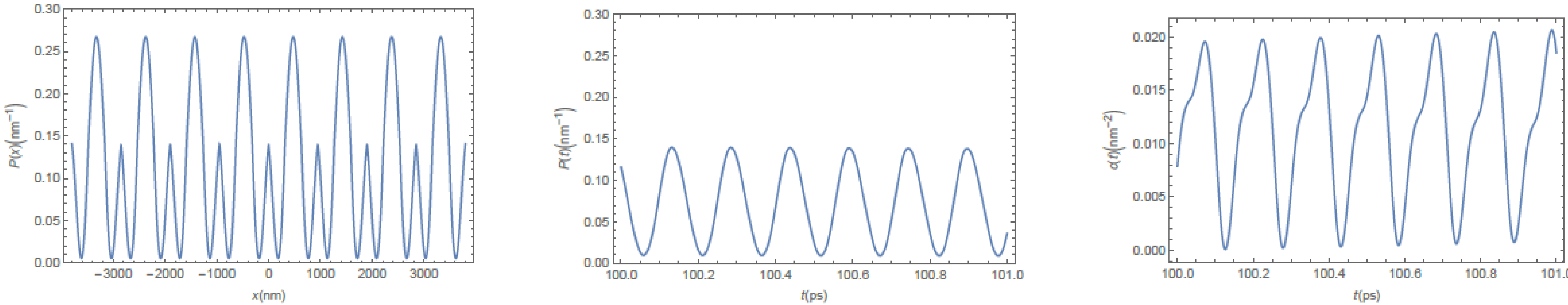


Figure 8: Two-point correlation function $c(t)$ as defined by Eq. (5) for the exciton-polariton condensate, with the addition of quantum noise. Like in Fig. 6, we considered the observable O to be the deviation between the condensate density $P(x,t)$ and the unperturbed steady-state density P_0. We took $x - x'$ to be half the length of the strip and calculated the expectation value $\left\langle \hat{O}(\mathbf{r},t)\hat{O}(\mathbf{r}',t')\right\rangle$ by taking the average value for all x while maintaining $x - x'$ constant.

Exciton BEC Density and Correlation Function in a twisted TMDC bilayer

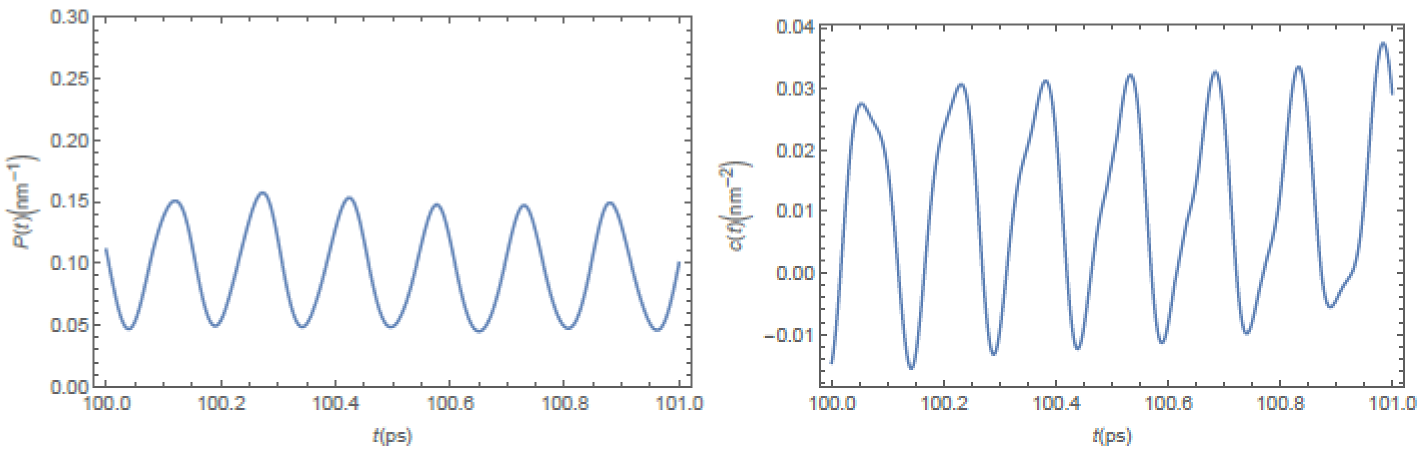
we considered the length of the strip to be $L_x = 4,000$ μm, and its width to be $L_y = 1$ μm.

$WSe_2/MoSe_2$ bilayer heterostructure twisted by 1°



(a) (b) (c)

Figure 9: Numerical results for the dynamics of the excitonic condensatein a twisted TMDC bilayer in the mean-field approach. (a) time evolution of the condensate density at position $x = 100$ nm; (b) Condensate density throughout the entire strip at $t = 3$ ns and (c) two-point correlator as defined in Eq. (5).



(a) (b)

Figure 10: Numerical results for the dynamics of the excitonic condensate in a twisted TMDC bilayer beyond the mean-field approach. (a) Condensate density in an arbitrary position of the strip. (b) Two-point correlator defined

G. P. Martins, O. L. Berman, and G. Gumbs, *Sci. Rep.* 13, 19707 (2023).

Conclusion

- Condensates of excitons and of microcavity polaritons in an external periodic potential obey the Semiclassical Time Crystal criterion
- Non-trivial correlations in the order parameter are seen even when quantum corrections are included in the Gross-Pitaevskii equation that governs the dynamics
- BECs of excitons have been achieved at temperatures of ~ 190 K, and BECs of microcavity polaritons have been achieved at room temperature. For this reason, we believe our proposed systems are good candidates for TCs in settings easily achievable in most labs.

I am very thankful to my collaborators:

Collaborators:

Godfrey Gumbs (Hunter College of CUNY)

Roman Kezerashvili (City Tech, CUNY)

Yurii Lozovik (Institute of Spectroscopy)

Klaus Ziegler (University of Augsburg, Germany)

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