Superfluidity in two-dimensional exciton bilayers system

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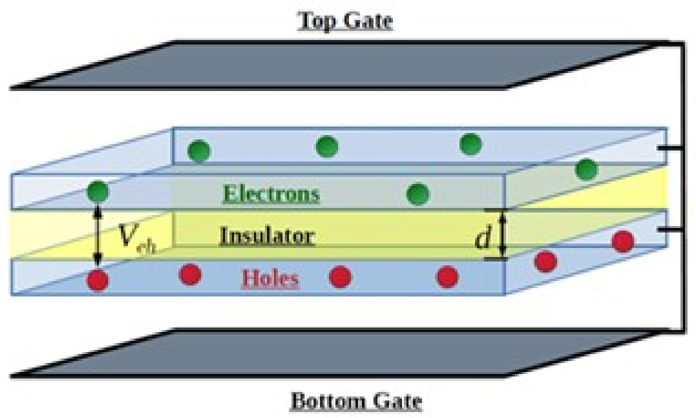
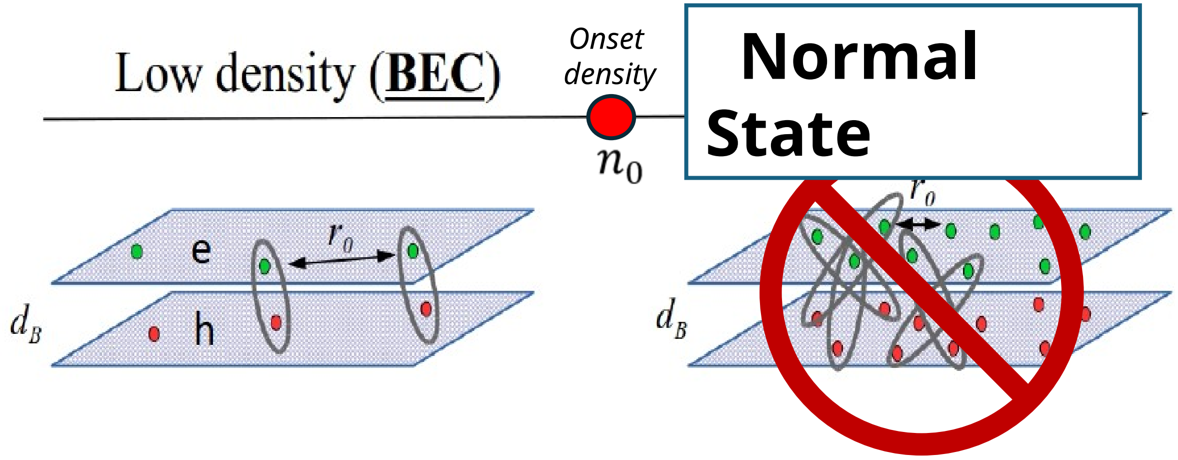
SCCS 2025
29/07/2025

Exciton bilayers





Screening kills exciton superfluidity!

Bare interactions:

$$V_q^S = \frac{2\pi e^2}{4\pi q}, \quad V_q^D = -\frac{2\pi e^2 e^{-qd}}{4\pi q}$$

Gap energy equation:

$$\Delta(k) = -\frac{1}{S}\sum_{k'} V_q^S\,(k-k')\,\frac{\Delta(k')}{2E(k')}$$

$$H = \sum_{\mathbf{k},\sigma} \xi_{\mathbf{k}}^{\sigma}\left(c^{\dagger}_{\mathbf{k},\sigma}c_{\mathbf{k},\sigma} + d^{\dagger}_{\mathbf{k},\sigma}d_{\mathbf{k},\sigma}\right) + H_{int}$$

$$H_{int} = \frac{1}{A}\sum_{\substack{\mathbf{k},\mathbf{k}',\mathbf{q}\\ \sigma,\sigma'}}\left[\frac{V^S_{\mathbf{q}}}{2}\left(c^{\dagger}_{\mathbf{k}+\mathbf{q},\sigma}c^{\dagger}_{\mathbf{k}'\mathbf{q},\sigma'}c_{\mathbf{k}',\sigma'}c_{\mathbf{k},\sigma} + d^{\dagger}_{\mathbf{k}+\mathbf{q},\sigma}d^{\dagger}_{\mathbf{k}'-\mathbf{q},\sigma'}d_{\mathbf{k}',\sigma'}d_{\mathbf{k},\sigma}\right) + V^D_{\mathbf{q}}c^{\dagger}_{\mathbf{k}+\mathbf{q},\sigma}d^{\dagger}_{\mathbf{k}'-\mathbf{q},\sigma'}d_{\mathbf{k}',\sigma'}c_{\mathbf{k},\sigma}\right]$$

Screening

Dyson equation:

$$\mathbf{W}(\mathrm{q},\omega) = \mathbf{V}(\mathrm{q}) + \mathbf{V}(\mathrm{q})\mathbf{\Pi}^*(q,\omega)\boldsymbol{W}(q,\omega)$$

$$\boldsymbol{W}(q,\omega) = \begin{pmatrix} V_{sc}^S(q,\omega) & V_{sc}^D(q,\omega) \\ V_{sc}^D(q,\omega) & V_{sc}^S(q,\omega) \end{pmatrix}, \qquad \boldsymbol{V}(q) = \begin{pmatrix} V_q^S & V_q^D \\ V_q^D & V_q^S \end{pmatrix}, \qquad \mathbf{\Pi}^*(q,\omega) = \begin{pmatrix} \Pi^N(q,\omega) & \Pi^A(q,\omega) \\ \Pi^A(q,\omega) & \Pi^N(q,\omega) \end{pmatrix}$$

$$\Pi^N(q,\omega) = -\frac{ig}{\hbar A}\lim_{\eta\to 0}\int_{-\infty}^{\infty} d\tau\, e^{i(\omega-i\eta)\tau}\langle T[\rho_e\rho_e S]\rangle_c, \qquad \Pi^A(q,\omega) = -\frac{ig}{\hbar A}\lim_{\eta\to 0}\int_{-\infty}^{\infty} d\tau\, e^{i(\omega-i\eta)\tau}\langle T[\rho_e\rho_h S]\rangle_c$$

$$S = T\left[e^{-\frac{i}{\hbar}\int d\tau' H_1[\tau]}\right] = 1 \cancel{- \frac{i}{\hbar}\int d\tau' H_1[\tau] + \cdots}$$

RPA

$$\mathbf{W}^{\boldsymbol{RPA}}(\mathrm{q},\omega) = \mathbf{V}(\mathrm{q}) + \mathbf{V}(\mathrm{q})\mathbf{\Pi_0}(q,\omega)\boldsymbol{W}^{\boldsymbol{RPA}}(q,\omega) \qquad \mathbf{\Pi_0}(q,\omega) = \begin{pmatrix} \Pi_0^N(q,\omega) & \Pi_0^A(q,\omega) \\ \Pi_0^A(q,\omega) & \Pi_0^N(q,\omega) \end{pmatrix}$$

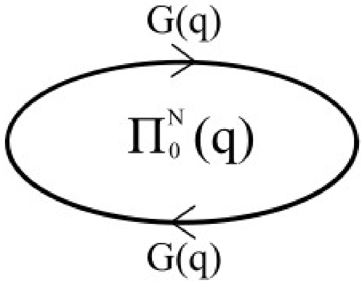
$$\Pi_0^N(q,\omega) = -\frac{ig}{\hbar A}\lim_{\eta\to 0}\int_{-\infty}^{\infty} d\tau\, e^{i(\omega-i\eta)\tau}\langle T[\rho_e\rho_e]\rangle_c, \qquad \Pi_0^A(q,\omega) = -\frac{ig}{\hbar A}\lim_{\eta\to 0}\int_{-\infty}^{\infty} d\tau\, e^{i(\omega-i\eta)\tau}\langle T[\rho_e\rho_h]\rangle_c$$

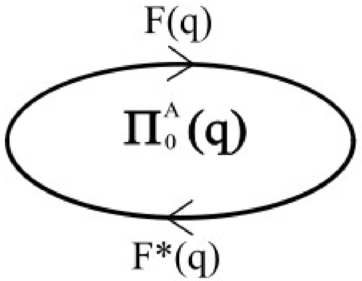
Screening

In the static limit $\omega \to 0$,

$$\Pi_0^N(\mathbf{q}) = \frac{ig}{\hbar A}\sum_{\mathbf{k}} \lim_{\eta,\omega\to 0}\int d\tau e^{i(\omega\tau + i\eta|\tau|)} iG_{\mathbf{k}+\mathbf{q}}(\tau) iG_{\mathbf{k}}(-\tau)$$
$$= -\frac{g}{A}\sum_{\mathbf{k}} \frac{u_{\mathbf{k}}^2 v_{\mathbf{k}+\mathbf{q}}^2 + v_{\mathbf{k}}^2 u_{\mathbf{k}+\mathbf{q}}^2}{E_{\mathbf{k}} + E_{\mathbf{k}+\mathbf{q}}}\,, \qquad (22)$$

$$\Pi_0^A(\mathbf{q}) = -\frac{ig}{\hbar A}\sum_{\mathbf{k}} \lim_{\eta,\omega\to 0}\int d\tau e^{i(\omega\tau + i\eta|\tau|)} iF^*_{\mathbf{k}+\mathbf{q}}(\tau) iF_{\mathbf{k}}(-\tau)$$
$$= -\frac{2g}{A}\sum_{\mathbf{k}} \frac{u_{\mathbf{k}} v_{\mathbf{k}} v_{\mathbf{k}+\mathbf{q}} u_{\mathbf{k}+\mathbf{q}}}{E_{\mathbf{k}} + E_{\mathbf{k}+\mathbf{q}}}\,. \qquad (23)$$





How reliable is RPA for exciton superfluidity?

- Tuning the density the system goes from dipolar (low density) to coulombic (high density) interactions.

Presence of superfluidity.

In standard superconductors (Eliashberg theory) significant first-order corrections have big effects.

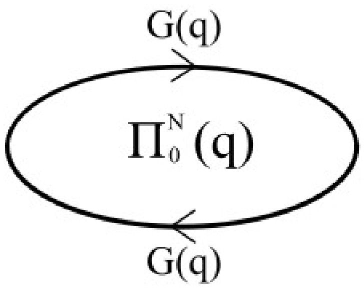
No Migdal’s theorem so exchange vertex corrections cannot a priori be neglected.

Screening

In the static limit $\omega \to 0$,

$$\Pi_0^N(\mathbf{q}) = \frac{ig}{\hbar A}\sum_{\mathbf{k}} \lim_{\eta,\omega\to 0}\int d\tau\, e^{i(\omega\tau+i\eta|\tau|)} iG \ldots \qquad \Pi_0^A(\mathbf{q}) = -\frac{ig}{\ldots}\sum \lim \int d\tau\, e^{i(\omega\tau+i\eta|\tau|)} iF^*_{\mathbf{k}+\mathbf{q}}(\tau) iF_{\mathbf{k}}(-\tau)$$

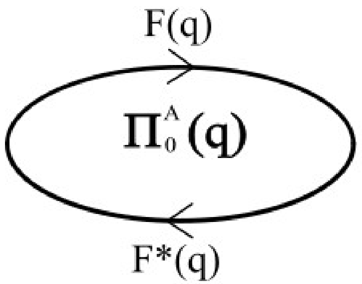
$$= -\frac{g}{A}\sum_{\mathbf{k}} \frac{u_{\mathbf{k}}^2 v_{\mathbf{k}+\mathbf{q}}^2 + v \ldots}{E_{\mathbf{k}} + E_{\mathbf{k}} \ldots} \qquad \ldots \frac{\ldots u_{\mathbf{k}+\mathbf{q}}}{\ldots{}_{\mathbf{k}+\mathbf{q}}}\,. \tag{23}$$



$$V_{\mathbf{q}}^{S,RPA} = \frac{V_{\mathbf{q}}^S - \Pi_0^N(\mathbf{q})\mathcal{A}_{\mathbf{q}}}{1 - 2(\Pi_0^N(\mathbf{q})V_{\mathbf{q}}^S + \Pi_0^A(\mathbf{q})V_{\mathbf{q}}^D) + \mathcal{B}_{\mathbf{q}}\mathcal{A}_{\mathbf{q}}}\,, \tag{24}$$

$$V_{\mathbf{q}}^{D,RPA} = \frac{V_{\mathbf{q}}^D + \Pi_0^A(\mathbf{q})\mathcal{A}_{\mathbf{q}}}{1 - 2(\Pi_0^N(\mathbf{q})V_{\mathbf{q}}^S + \Pi_0^A(\mathbf{q})V_{\mathbf{q}}^D) + \mathcal{B}_{\mathbf{q}}\mathcal{A}_{\mathbf{q}}}\,, \tag{25}$$

with $\mathcal{A}_{\mathbf{q}} = (V_{\mathbf{q}}^S)^2 - (V_{\mathbf{q}}^D)^2$ and $\mathcal{B}_{\mathbf{q}} = (\Pi_0^N(\mathbf{q}))^2 - (\Pi_0^A(\mathbf{q}))^2$.



How reliable is RPA for exciton superfluidity?

- Tuning the density the system goes from dipolar (low density) to coulombic (high density) interactions.

Presence of superfluidity.

In standard superconductors (Eliashberg theory) significant first-order corrections have big effects.

No Migdal's theorem so exchange vertex corrections cannot a priori be neglected.

Beyond RPA

Gaetano Senatore (SISSA), Trieste, Italy

Stefania de Palo (SISSA, UniTS), Trieste, Italy

SISSA Scuola Internazionale Superiore di Studi Avanzati

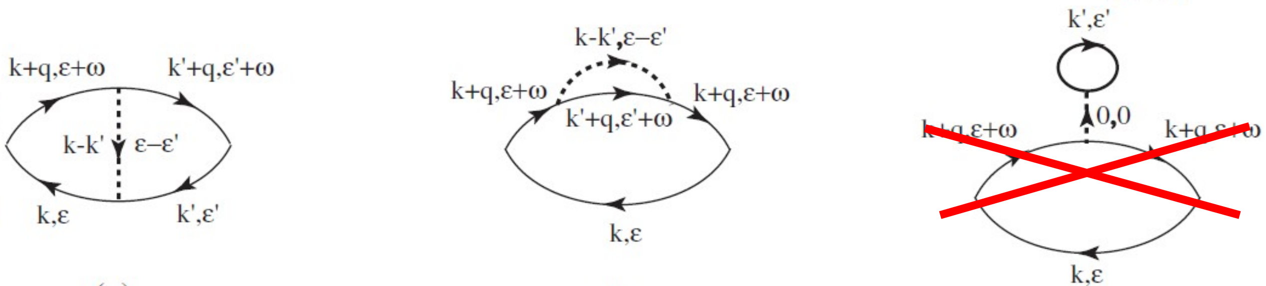
At fixed d, by tuning the density when do first order-corrections to the polarization function become important?

$$\Pi^N(\mathbf{q},\omega) = \Pi_0^N(\mathbf{q},\omega) + \Pi_1^N(\mathbf{q},\omega)\ ,$$
$$\Pi^A(\mathbf{q},\omega) = \Pi_0^A(\mathbf{q},\omega) + \Pi_1^A(\mathbf{q},\omega)\ ,$$

$$\Pi_1^N(\mathbf{q},\omega) = -\frac{g}{2\hbar^2 A}\lim_{\eta\to 0}\iint d\tau d\tau' e^{i(\omega\tau + i\eta|\tau|)}\langle T[\rho_e\rho_e H_1(\tau')]\rangle_c\ , \tag{28}$$

$$\Pi_1^A(\mathbf{q},\omega) = -\frac{g}{2\hbar^2 A}\lim_{\eta\to 0}\iint d\tau d\tau' e^{i(\omega\tau + i\eta|\tau|)}\langle T[\rho_e\rho_h H_1(\tau')]\rangle_c\ . \tag{29}$$

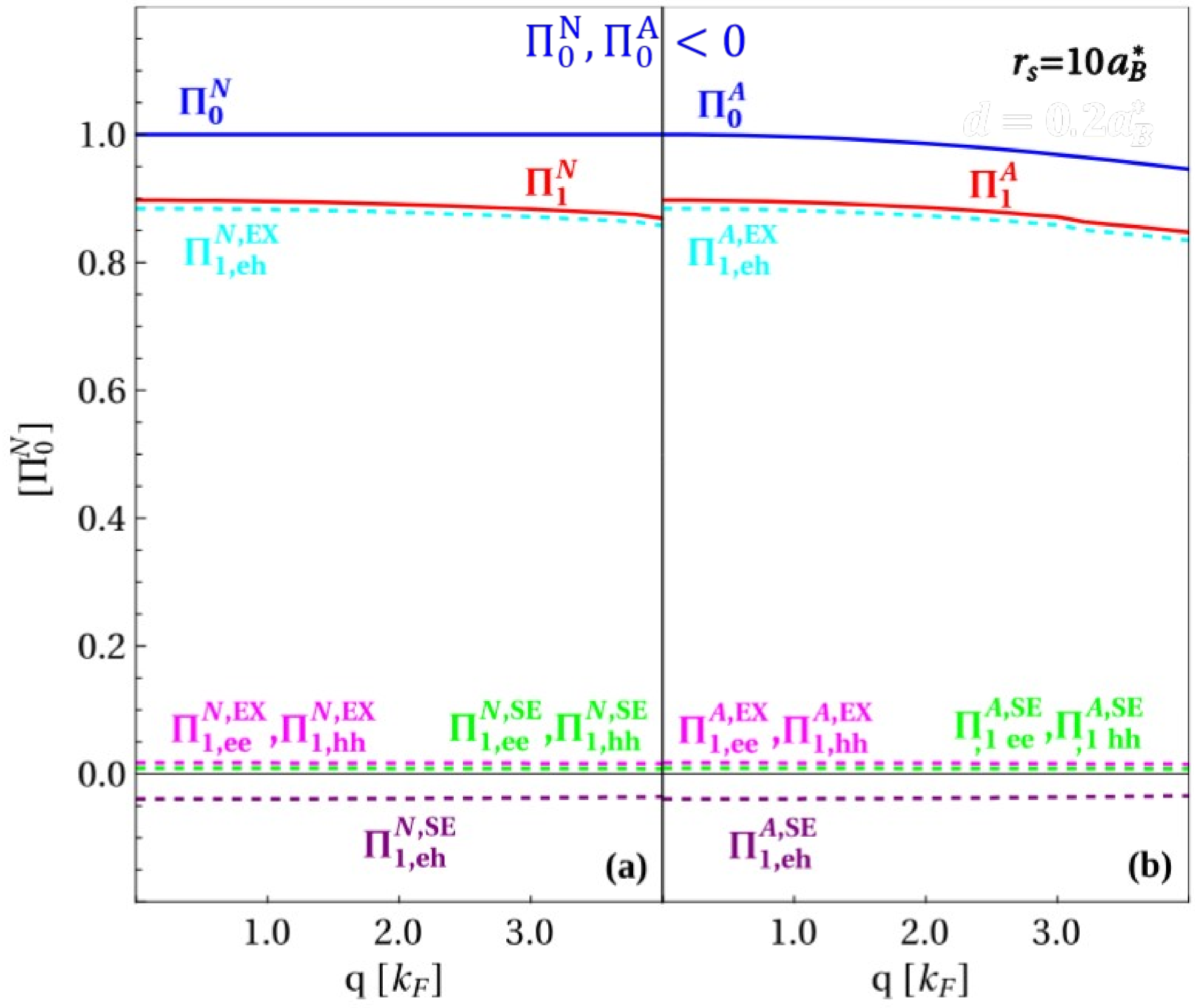
$$\Pi_1^N(\mathbf{q},\omega) = \Pi_{1,ee}^N(\mathbf{q},\omega) + \Pi_{1,hh}^N(\mathbf{q},\omega) + \Pi_{1,eh}^N(\mathbf{q},\omega)\ ,$$
$$\Pi_1^A(\mathbf{q},\omega) = \Pi_{1,ee}^A(\mathbf{q},\omega) + \Pi_{1,hh}^A(\mathbf{q},\omega) + \Pi_{1,eh}^A(\mathbf{q},\omega)\ ,$$

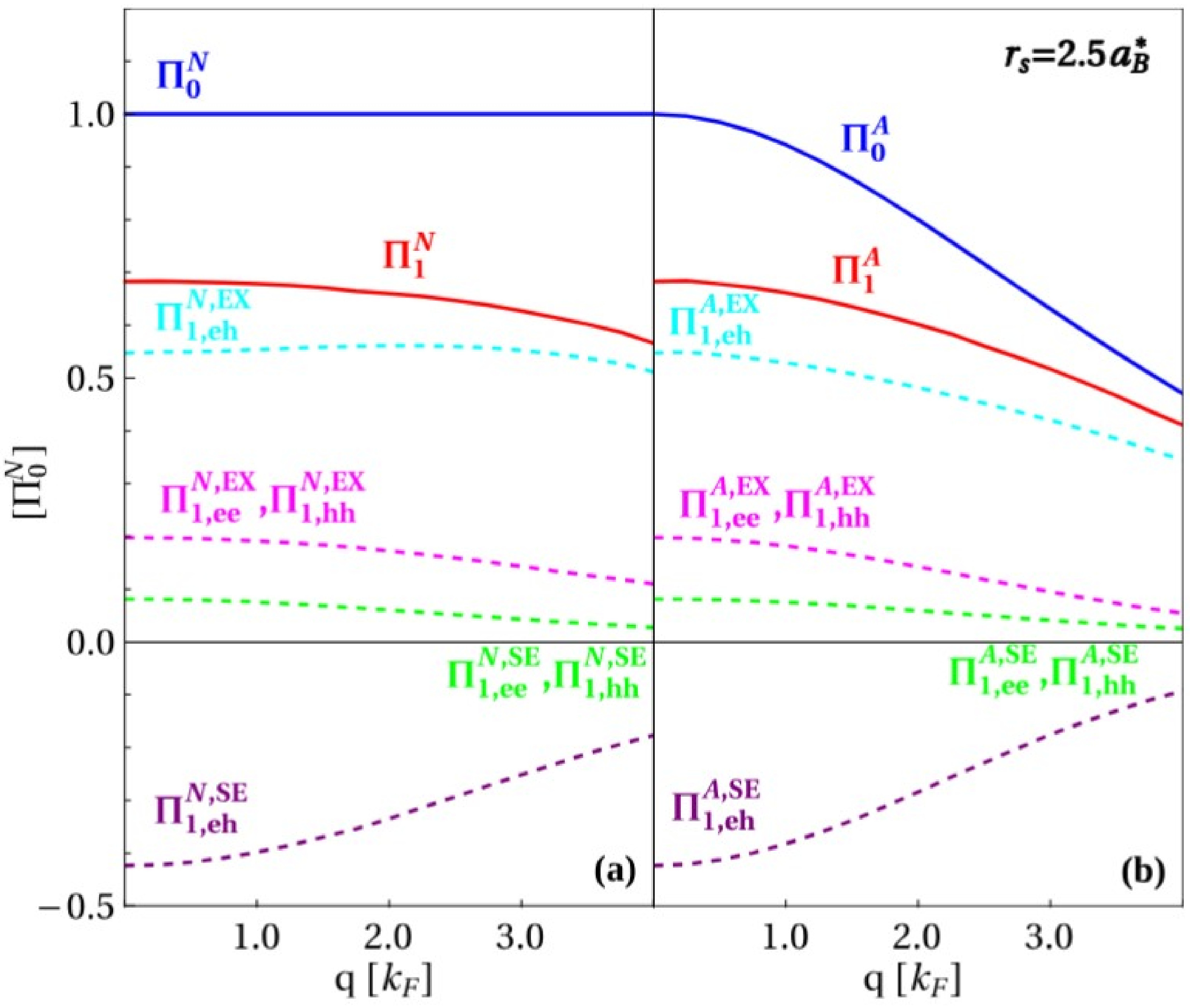


(a) (b) (c)

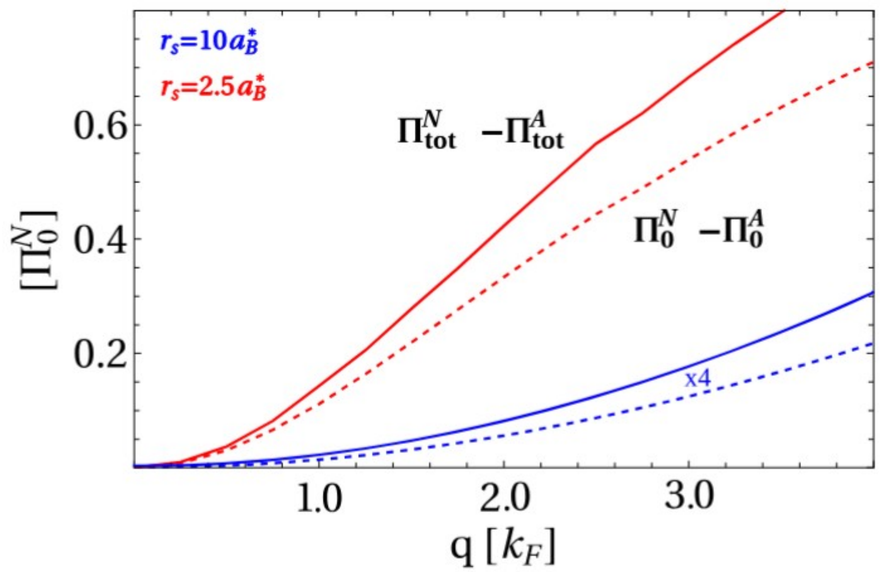
Charge neutrality

First-order corrections



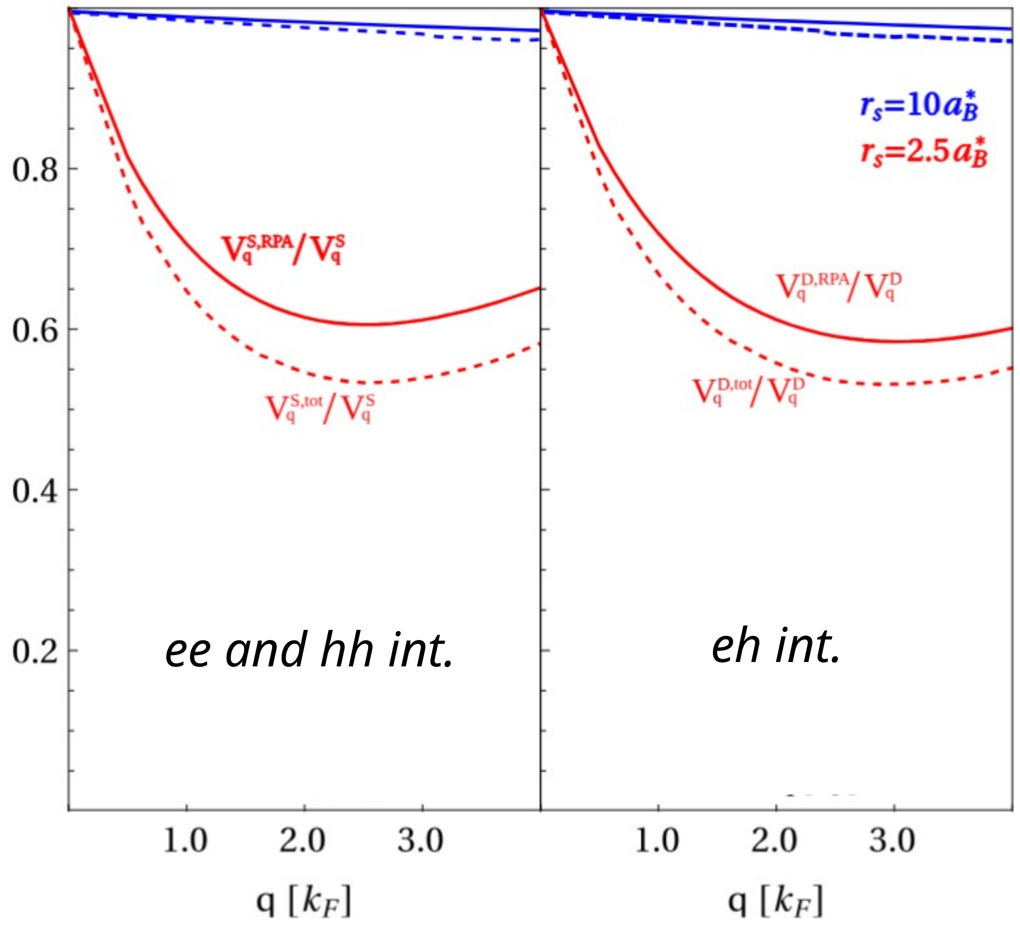


Beyond RPA screened interactions



- **Small density:** the zero-order (RPA) and first-order screened interactions match the bare interaction,

- **High density:** the RPA interaction is reduced by 30% around k_F respect the bare one. The effect of the first-order terms is a further reduction.



The less effective $\Pi^N - \Pi^A$ at large momenta and high density does not translate in a strong effect to the screened interactions. At large momenta the states that participate to the screening are empty.

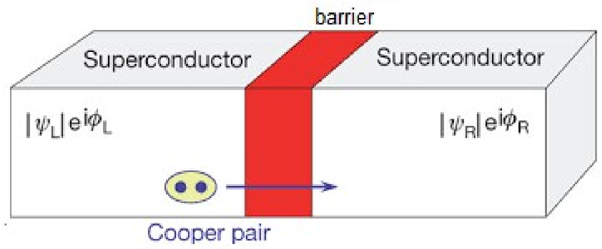
Conclusions: Beyond RPA

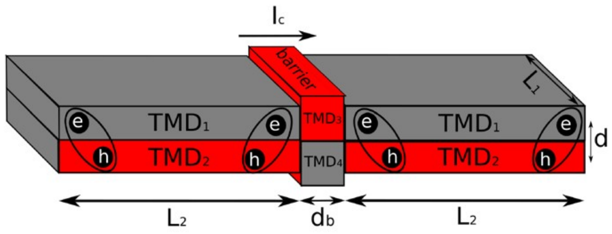
- **Small density:**
  - Strong cancellation between normal and anomalous polarization functions. The screened interactions are well approximated by the bare Coulomb interactions.
  - The electron-hole exchange term dominates over the other first-order corrections. In contrast with Migdal-Eliashberg theory for standard superconductors.

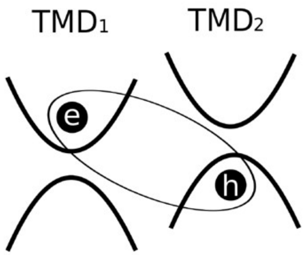
- **High density:**
  - Weak cancellation between normal and anomalous polarization functions *at high momenta*.
  - Little effect on the screened interactions.

The RPA works well for the exciton superfluid bilayer system across the full range of density.

Josephson effect in exciton bilayer systems







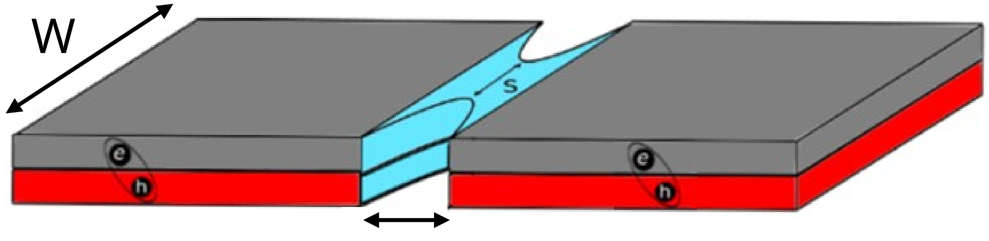
Motivations:
- Coherent dissipationless tunneling current is enough.
- Well known.
- Transport measurements.

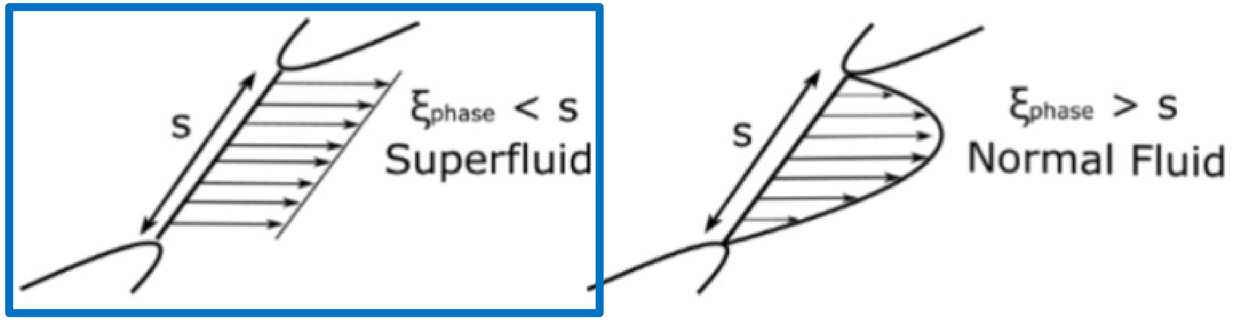
Issues:
- Fabrication of bilayer exciton Josephson junctions.
- Coherent excitonic current.
- Standard JE experiments are based on measurement of very low voltages and resistances.

Alex Hamilton (QED- UNSW), Sydney, Australia

Experimental Proposal

Fabrication of bilayer exciton Josephson junctions : **Exciton bilayer Dayem bridge.**



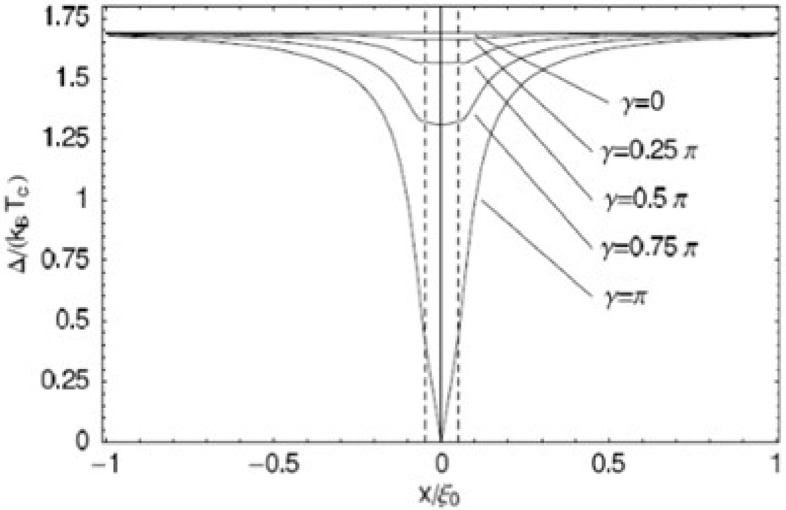


S-S'-S

$L_b < 3{,}5\xi_{phase} = 30\text{nm}$

S-N-S

$L_b < \xi_{phase} = 9\text{ nm}$

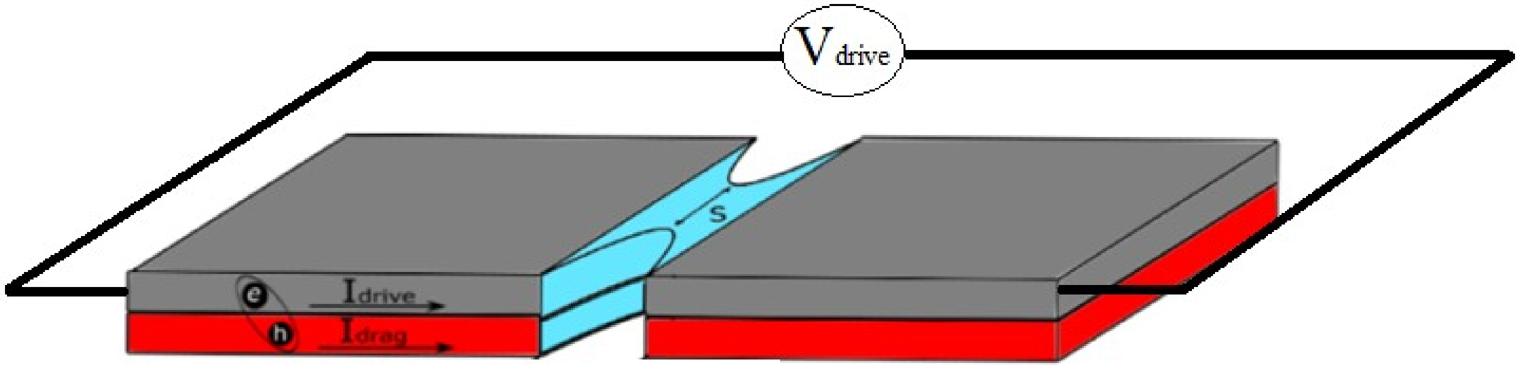


$$\gamma = \frac{mjWL_b}{\hbar ns}$$

A. Guman *et al.* Phys. Rev. B **76**, 064529 (2007).

Proposal

Injection of a coherent excitonic current : **Josephson Coulomb drag in exciton bilayer Dayem bridge.**



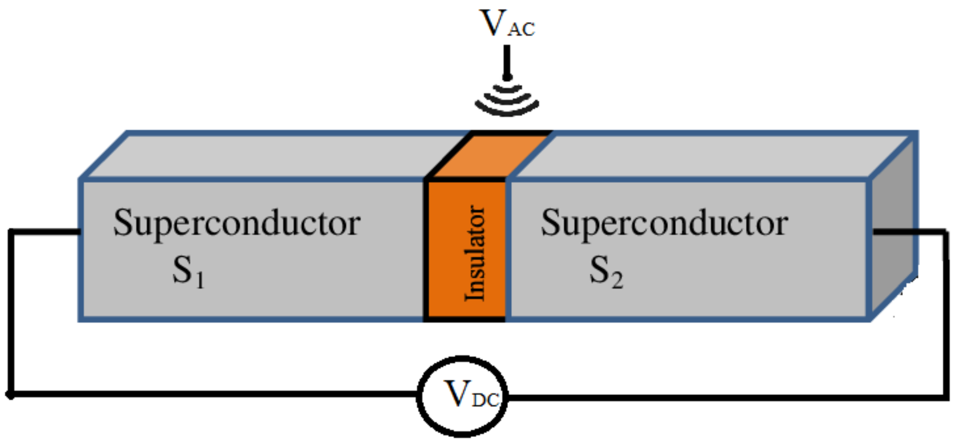
Injection of drive current in one layer. If the system is superfluid the drag is nearly perfect (Nguyen *et al.* Science 388-6744 (2025)).

BONUS: As a function of current, density and temperature we can also identify the transition from exciton system of electron-hole (decoupled) phase.

Proposal

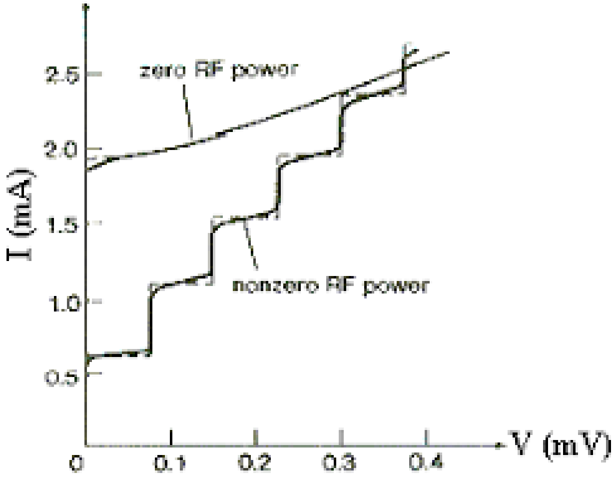
Measurements of low voltages and resistances:

Shapiro step in exciton bilayer Josephson Coulomb drag Dayem bridge.



$$V_{AC} = V_1 \cos(\omega t)$$

$$\phi(t) = \phi_0 + \frac{2etV_{DC}}{\hbar} + \frac{2e}{\omega\hbar} V_1 \sin(\omega t)$$



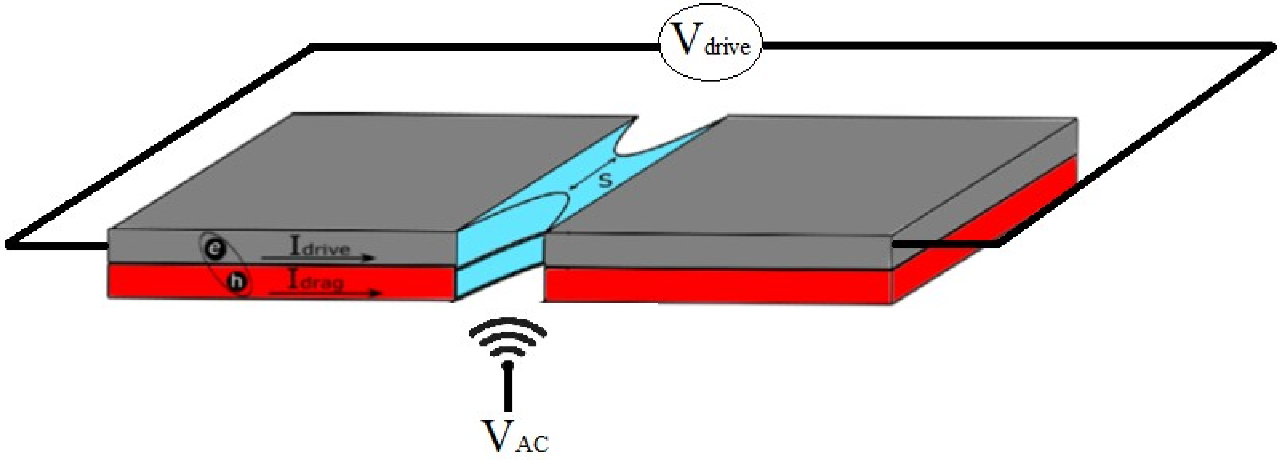
Steps every time:

$$V_{DC} = \frac{k\hbar\omega}{2e}, k = 1, 2, 3 \ldots$$

Proposal

Measurements of low voltages and resistances:

Shapiro step in exciton bilayer Josephson Coulomb drag Dayem bridge.



$$V(t) = V_{dc} + V_{ac} \cos \omega_1 t$$

$$\phi(t) = \int dt \frac{eV_{dc}}{2} + \vec{p}_e \cdot \vec{E}(r,t) + \vec{p}_h \cdot \vec{E}(r+d,t)$$

$$p_e = eL$$

$$p_h = -eL$$

- Angle of radiation.
- Not trivial dependence between potential and apperence of the steps.
- First time exploration of the Shapiro steps throughout the BCS-BEC crossover.

Collective Modes: density CM

Hadrien Kurkjian (CNRS, LPTMC), Sorbonne, Paris , France

RPA dressed polarization function:

$$\mathbf{\Pi}_{\boldsymbol{RPA}}(q,\omega) = \mathbf{\Pi}_{\mathbf{0}}^{*}(q,\omega)) * \left(\mathbb{1} - \boldsymbol{V}(q)\boldsymbol{\Pi}_{\mathbf{0}}^{*}(q,\omega)\right)^{-1}, \quad \boldsymbol{V}(q) = \begin{pmatrix} V_S(q) & V_D(q) \\ V_D(q) & V_S(q) \end{pmatrix}, \quad \mathbf{\Pi}_{\mathbf{0}}^{*}(q) = \begin{pmatrix} \Pi_0^N(q,\omega) & \Pi_0^A(q,\omega) \\ \Pi_0^A(q,\omega) & \Pi_0^N(q,\omega) \end{pmatrix}.$$

$$\boxed{\Pi_{RPA} = \frac{1}{1 - 2(V_S\Pi_0^N + V_D\Pi_0^A) + AB} \begin{pmatrix} \Pi_0^N - V_S B & \Pi_0^A + V_D B \\ \Pi_0^A + V_D B & \Pi_0^N - V_S B \end{pmatrix}}$$

$$A(q) = V_S^2(q) - V_D^2(q), \qquad B(q,\omega) = \Pi_0^N(q,\omega)^2 - \Pi_0^A(q,\omega)^2$$

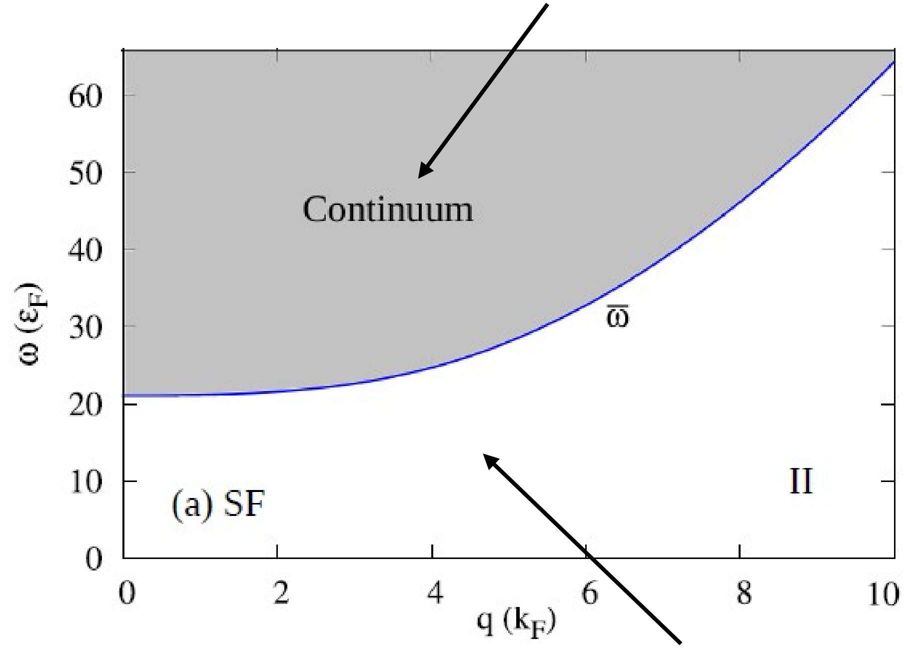
Energy spectra of the collective modes is given by the poles of $\Pi_{RPA}(q,\omega)$:

$$\boxed{1 - 2(V_S(q)\Pi_0^N(q,\omega) + V_D(q)\Pi_0^A(q,\omega)) + A(q)B(q,\omega) = 0}$$

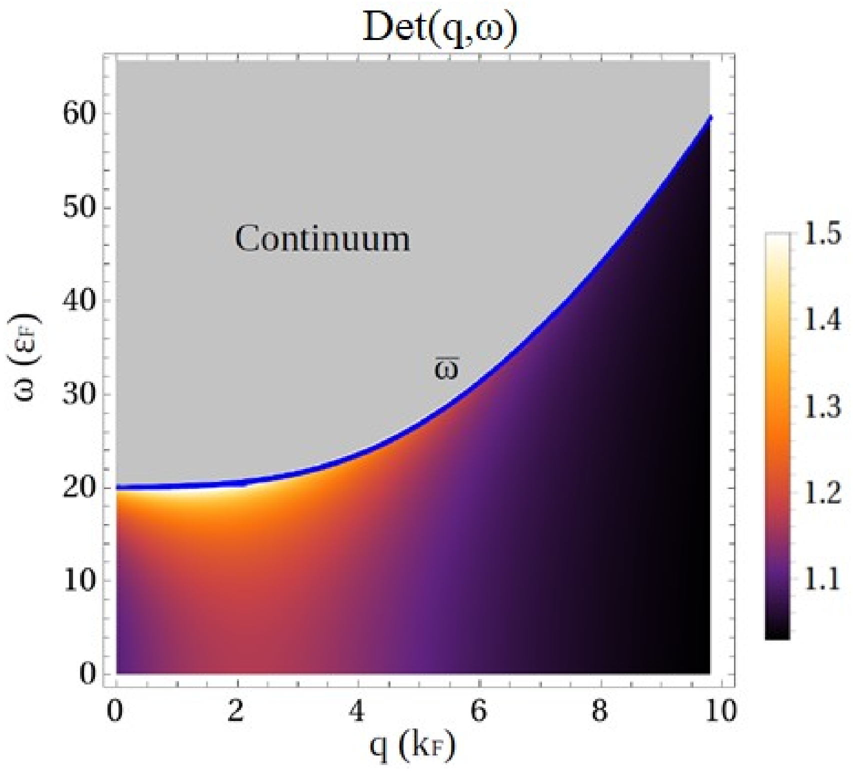
BEC regime ($n \ll n_0$)

$$1 - 2(V_S(q)\Pi_0^N(q,\omega) + V_D(q)\Pi_0^A(q,\omega)) + A(q)B(q,\omega) = Det(q,\omega) = 0$$

$\Pi_0^N(q,\omega)$ and $\Pi_0^A(q,\omega)$ diverge. The solution is complex. Damped collective modes.



$\Pi_0^N(q,\omega)$ and $\Pi_0^A(q,\omega)$ do not diverge. The solution is real. Undamped collective modes.



No stable density collective modes outside the continuum in the BEC.

BEC regime ($n \ll n_0$)

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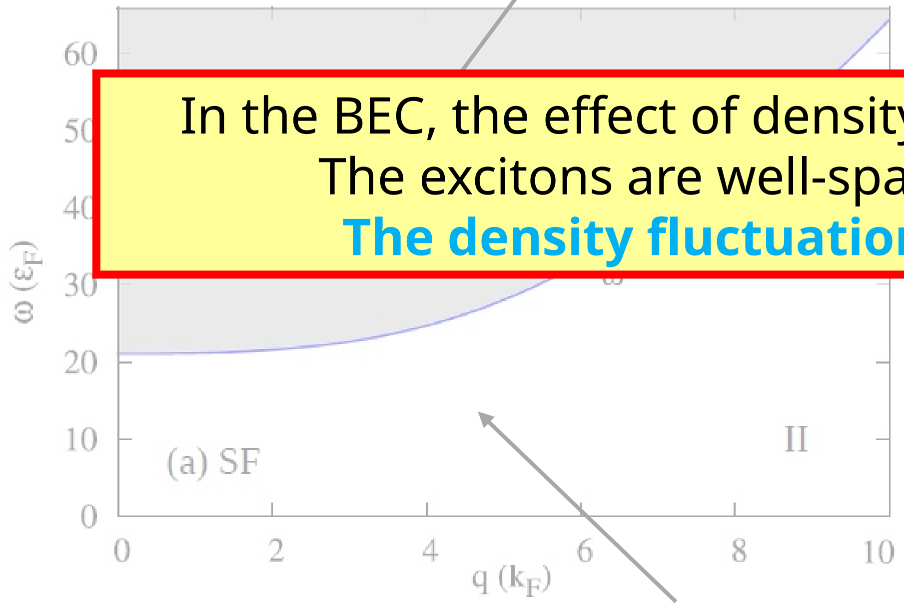
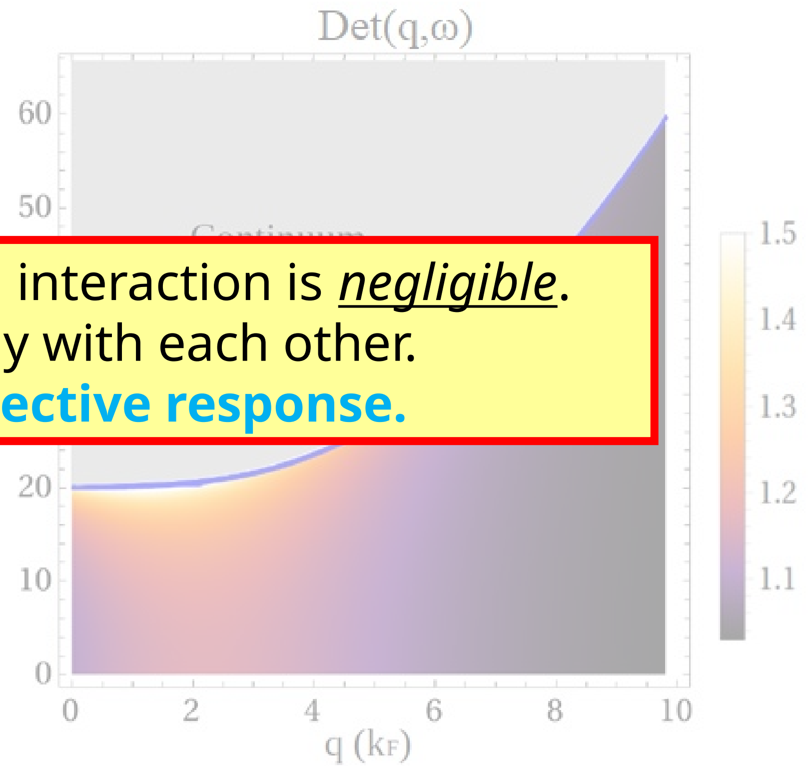


In the BEC, the effect of density fluctuations on the screening interaction is ***negligible***.
The excitons are well-spaced and they interact minimally with each other.
The density fluctuations do not induce a stable collective response.



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No stable density collective modes outside the continuum in the BEC.

Crossover regime ($n \lesssim n_0$)

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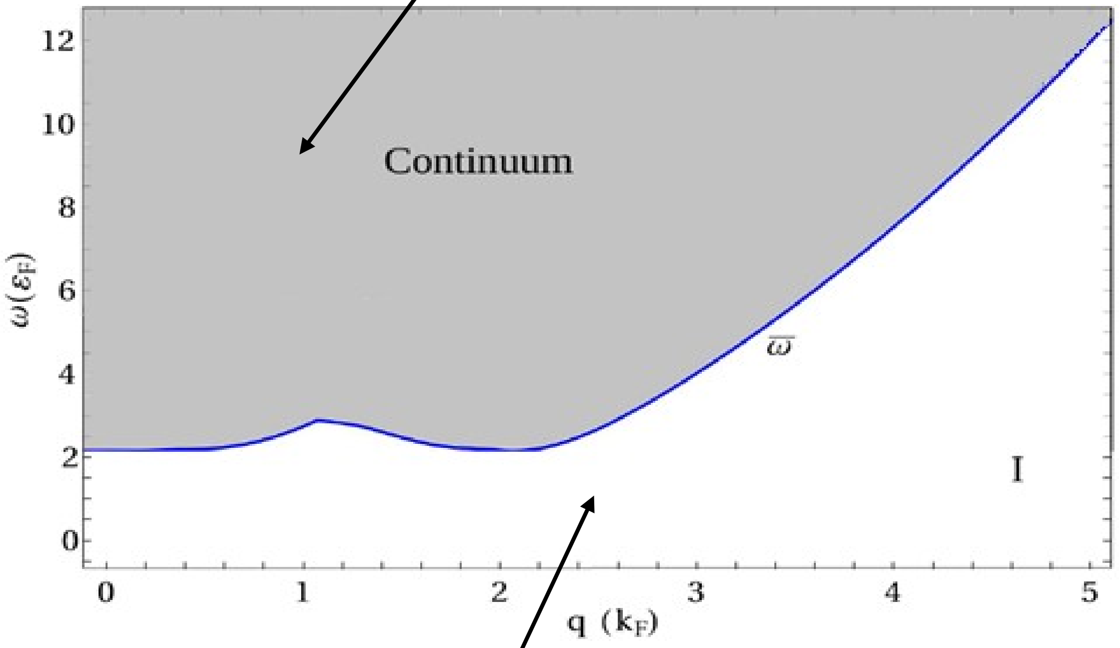

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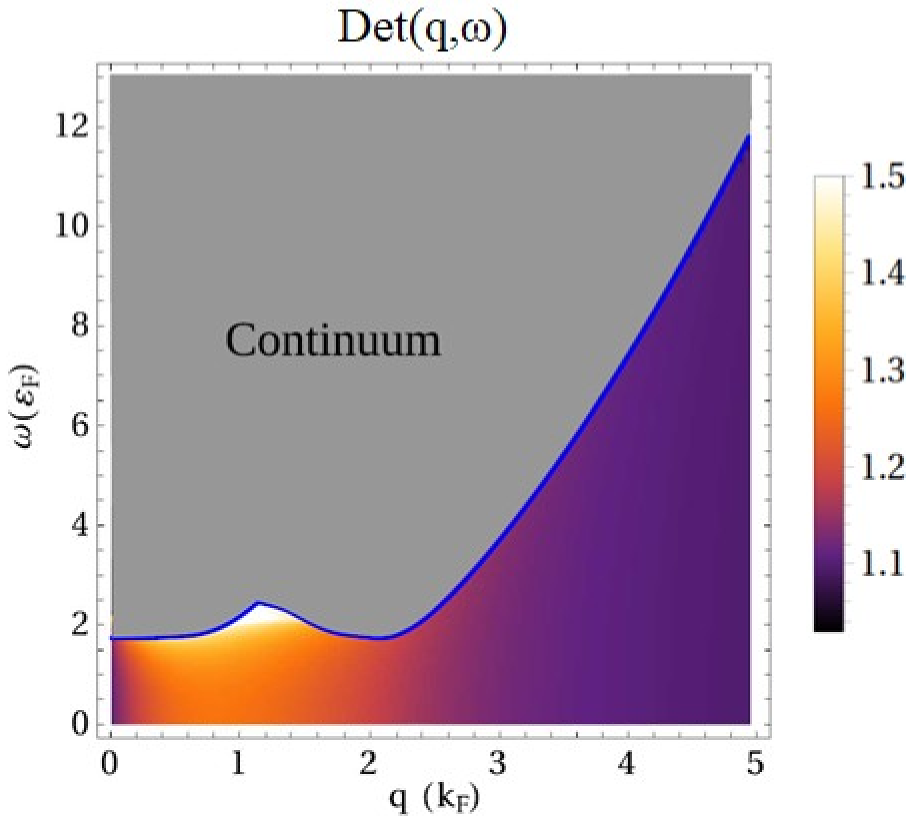

No stable density collective modes outside the continuum in the exciton superfluid phase.

Crossover regime ($n \lesssim n_0$)

$$1 - 2(V_S(q)\Pi_0^N(q,\omega) + V_D(q)\Pi_0^A(q,\omega)) + A(q)B(q,\omega) = Det(q,\omega) = 0$$

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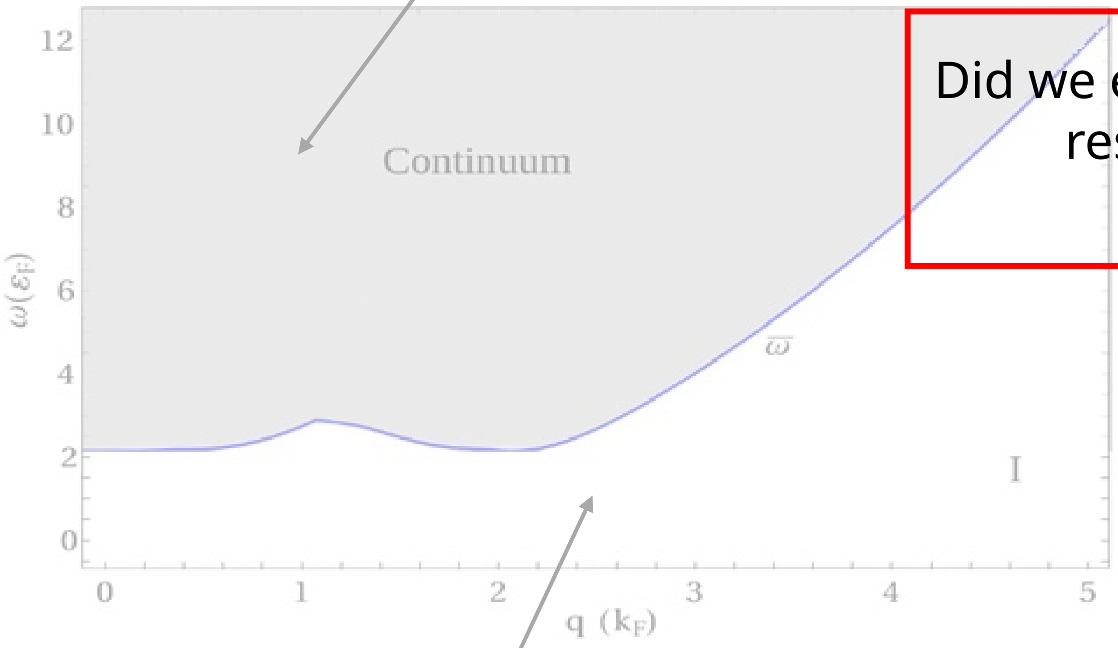
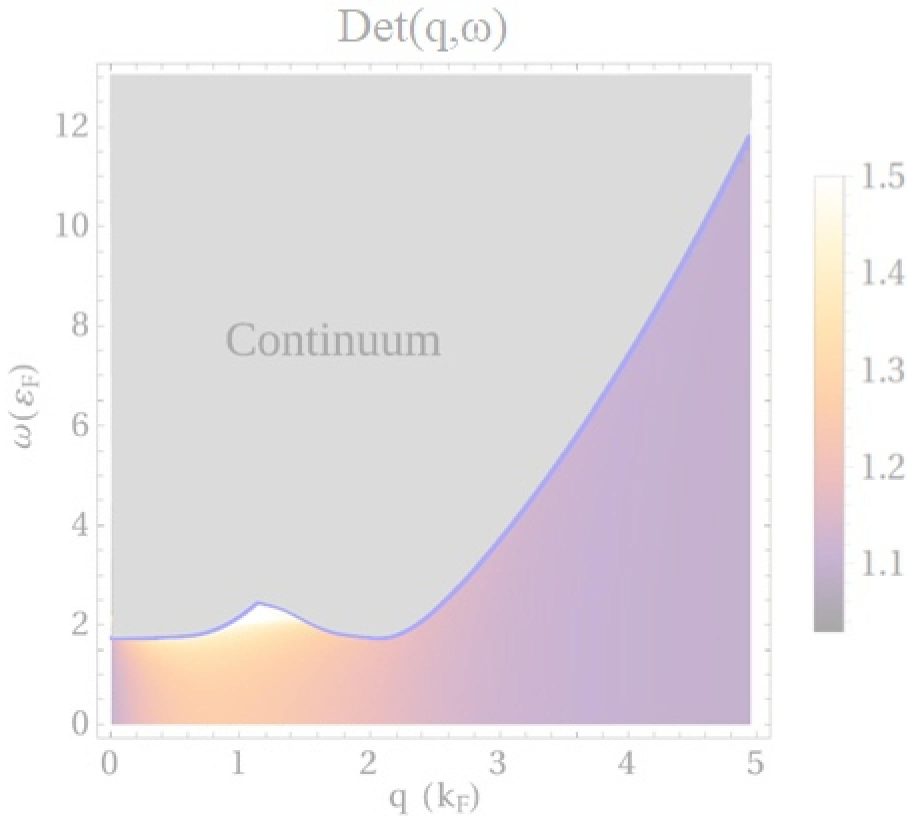


Did we expect this result?
No.



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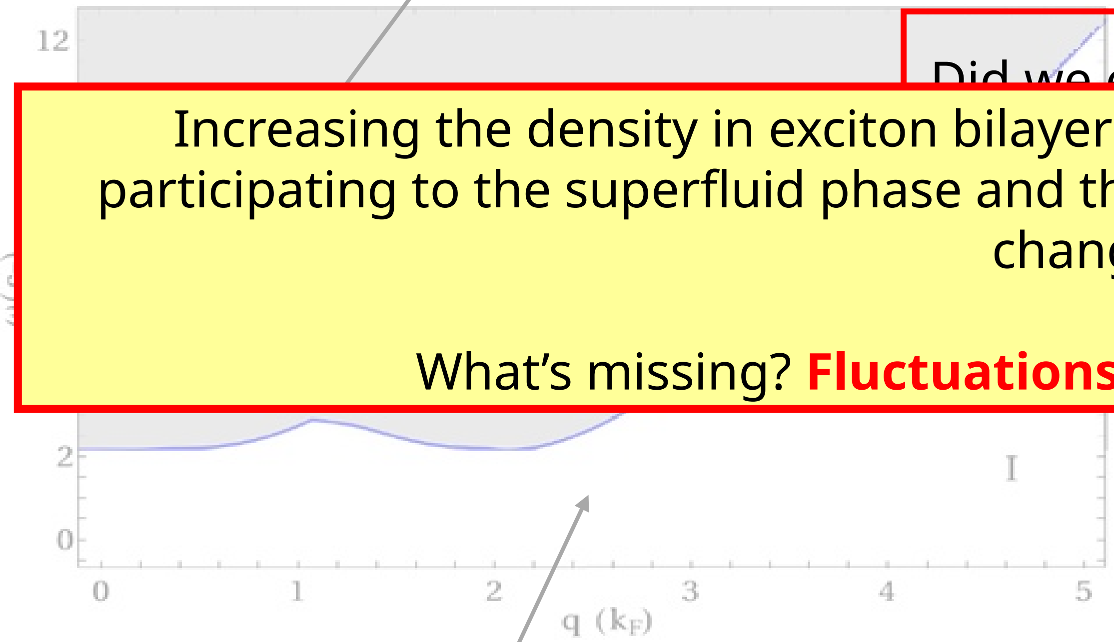
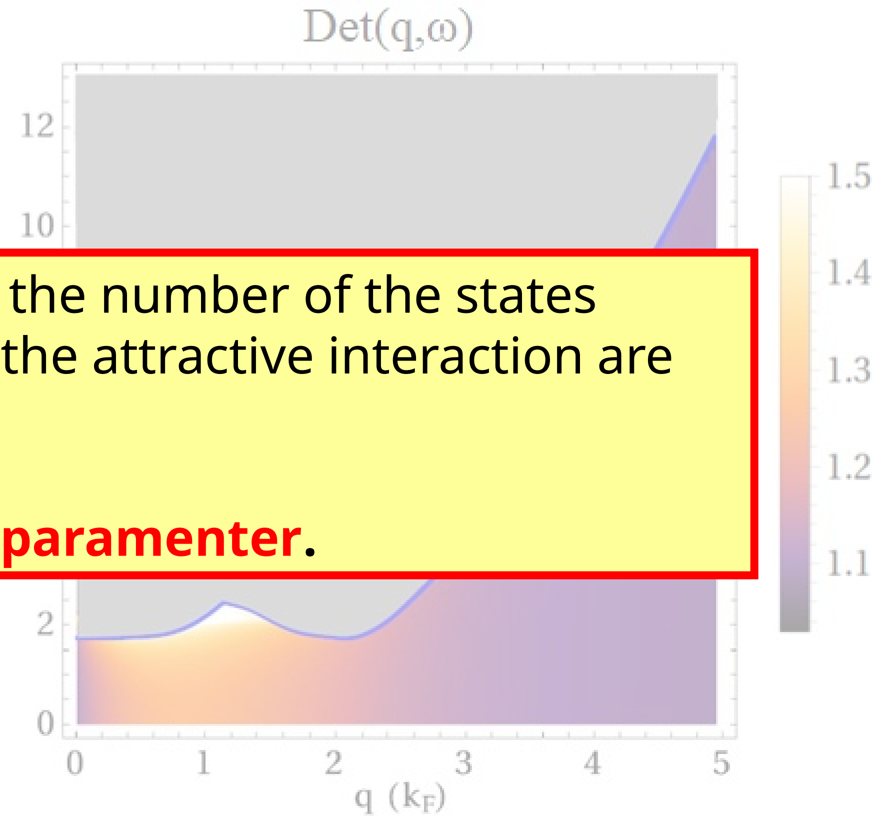


Did we expect this

Increasing the density in exciton bilayer systems both the number of the states participating to the superfluid phase and the strength of the attractive interaction are changing.

What's missing? **Fluctuations of the order paramenter**.



$\Pi_0^N(q,\omega)$ and $\Pi_0^A(q,\omega)$ do not diverge. The solution is real. Undamped collective modes.

No stable density collective modes outside the continuum in the exciton superfluid phase.

Normal State ($n \gtrsim n_0$)

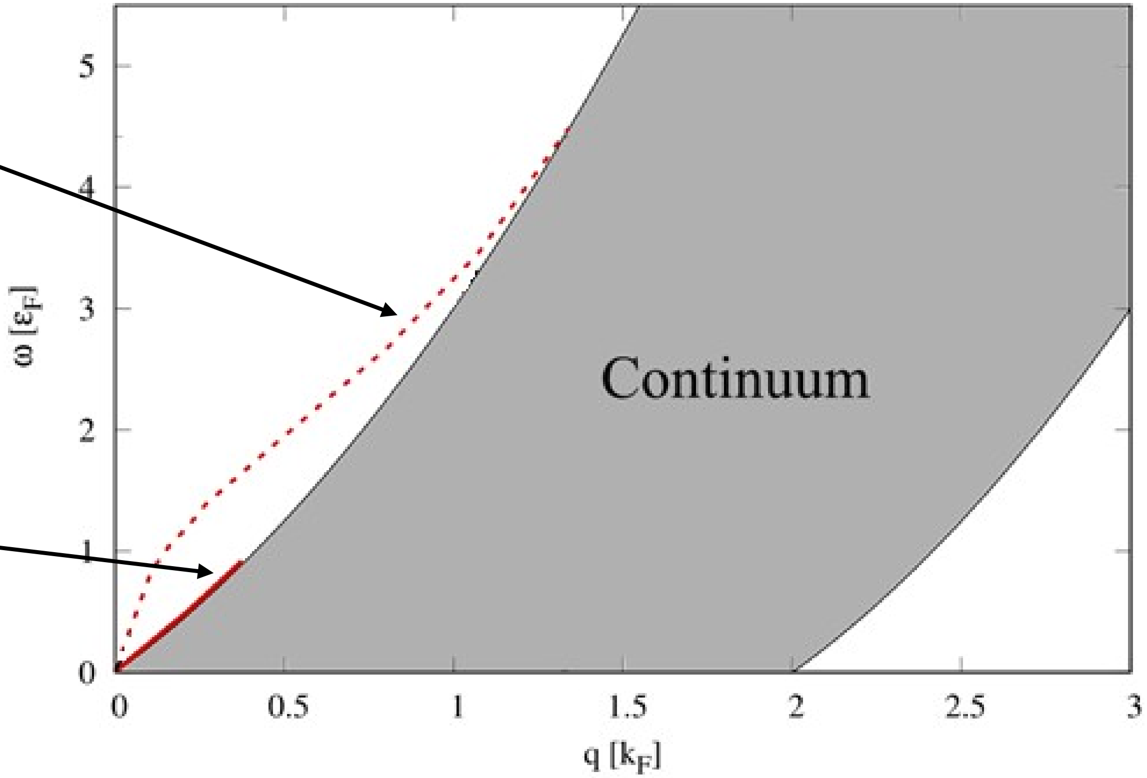
$$\boxed{1 - 2(V_S(q)\Pi_0^N(q,\omega) + V_D(q)\Pi_0^A(q,\omega)) + A(q)B(q,\omega) = Det(q,\omega) = 0}$$

In the normal state, $\Pi_0^A(q,\omega) = 0$.

$$\boxed{1 - 2V_S(q)\Pi_0^N(q,\omega) + A(q)\Pi_0^N(q,\omega) = Det(q,\omega) = 0}$$

Density *optic* mode:
The excitons oscillates
out-phase.

Density *acoustic* mode:
The excitons oscillates
in-phase.



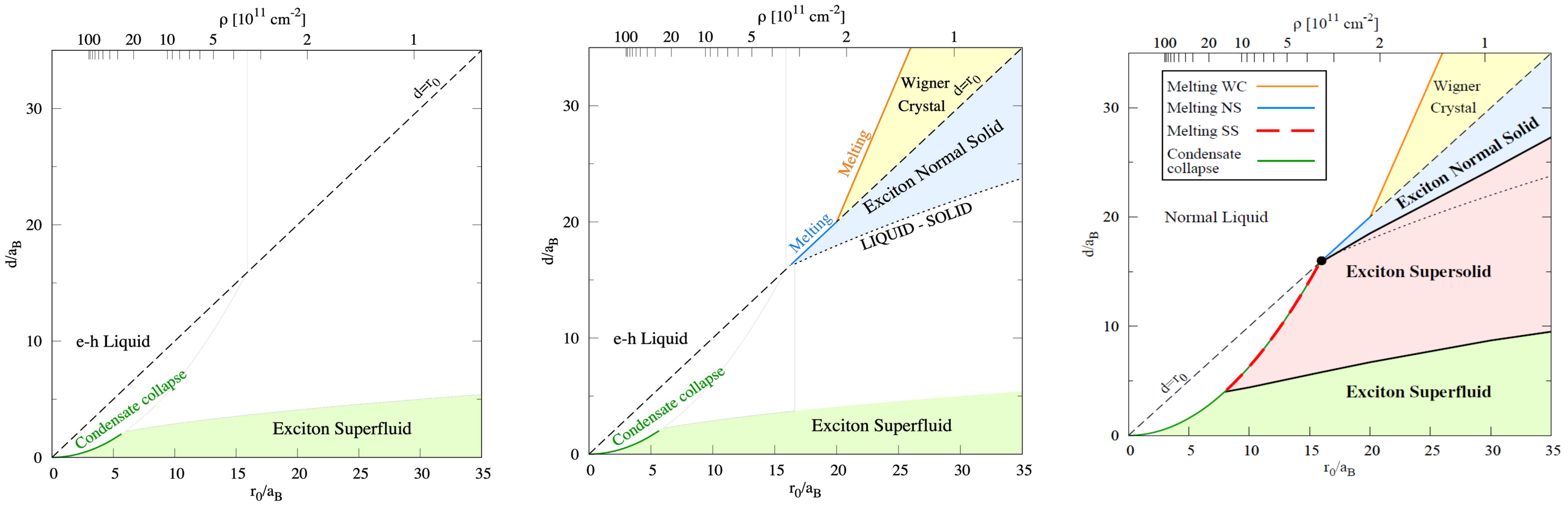
Normal state: *two* types of particles, *two* branches.

Superfluid state: *one* exction superfluid. Optic should disappears and the acoustic should remain

$$\boxed{\Delta > E_F}$$

$$\boxed{E_{ac.}^{CM} < \Delta}$$

Superfluidity and ....



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