

Excitonic superfluidity/supersolidity in electron-hole bilayer systems

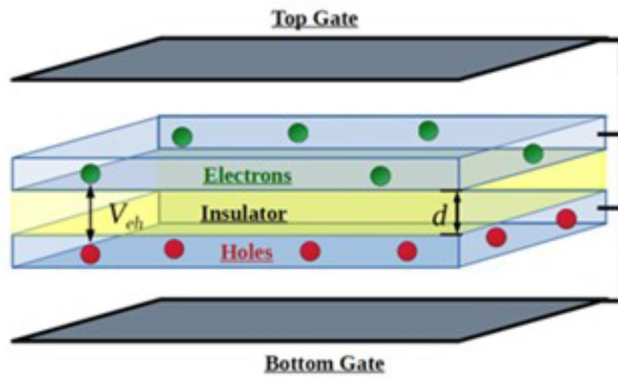
David Neilson



Sara Conti (Antwerp)
Filippo Pascucci (Antwerp)
Jacques Tempère (Antwerp)
Andrey Chaves (Fortaleza)
Alexander Hamilton (Sydney)
Milorad Milošević (Antwerp)
Luis Peña Ardila (Trieste)
Andrea Perali (Camerino)
Giordano Scappucci (Delft)



Research Foundation
Flanders
Opening new horizons



Strong Coulomb long-range electron-hole attraction $\rightarrow$ Electron-hole pairs $\rightarrow$ Quantum condensation and superfluidity $\rightarrow$ High Temperatures ?

L. V. Keldysh and Y. V. Kopayev *Possible instability of semimetallic state toward Coulomb interaction*

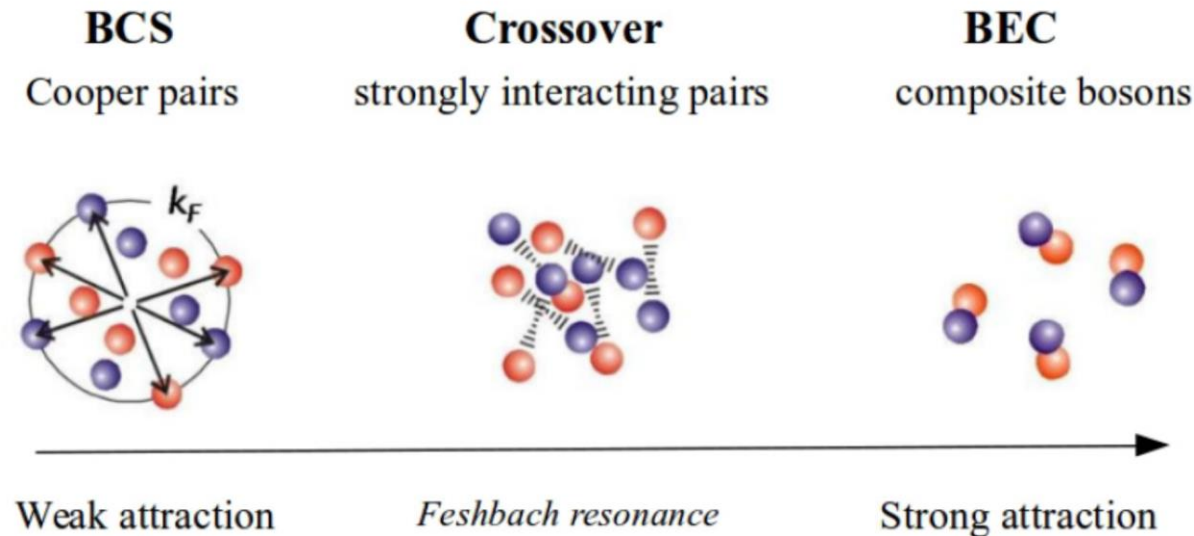
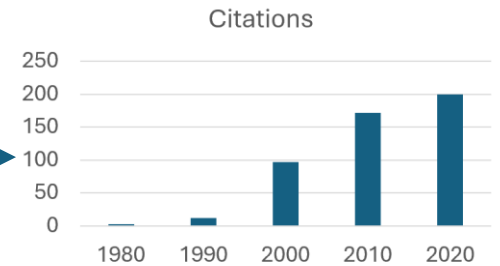
Sov. Phys. - Solid State **6** 2219 (1965) (Fizika Tverdogo Tela, **6**, 2791 (1964))

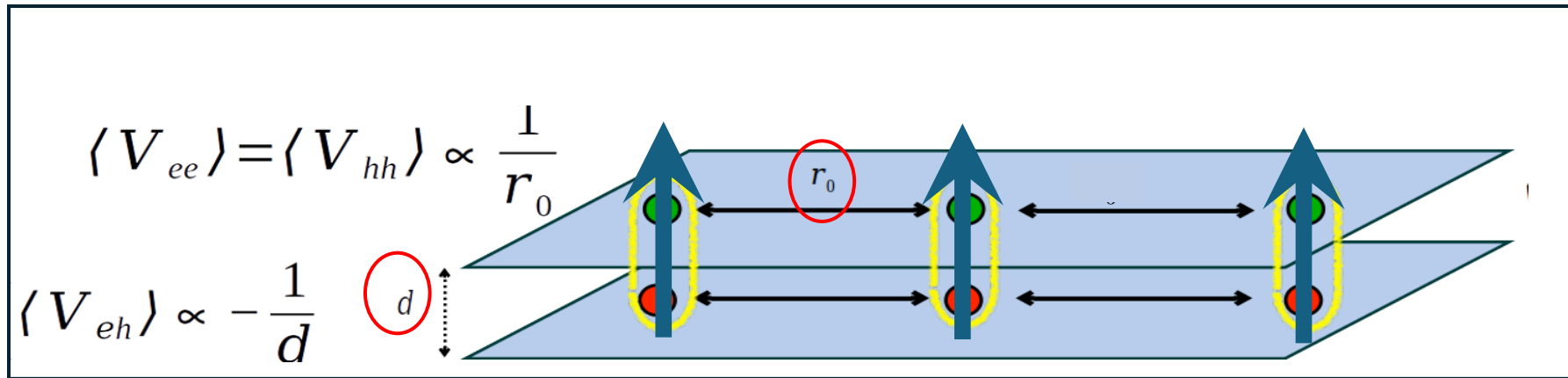
Y. E. Lozovik and V. Yudson *Feasibility of superfluidity of paired spatially separated electrons and holes*

JETP Lett. **22** 274 (1975) (Pis'ma Zh. Eksp. Teor. Fiz. **22**, 556 (1975))

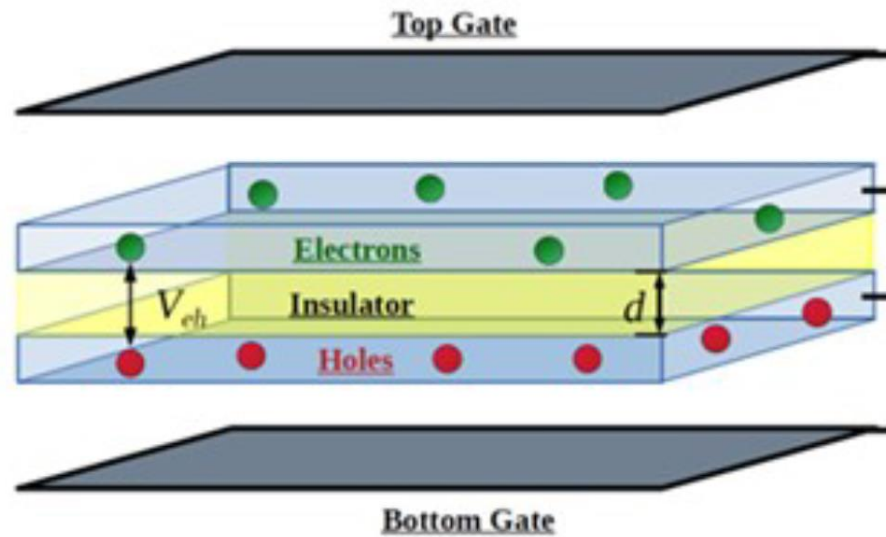
S. I. Shevchenko *Theory of superconductivity of systems with pairing of spatially separated electrons and holes*

Sov. J. Low Temp. Physics **2** 251 (1976) (Fiz. Nizk. Temp. **2**, 505 (1976))





- Exciton quasi-dipoles align perpendicular to the layers
- Small $d \Rightarrow$ strong binding but weak interactions between dipolar excitons
- Large $d \Rightarrow$ strong interactions between excitons
- Exciton-exciton interaction is **always repulsive** (unlike for cold atoms)



Exciton Superfluidity AT EQUILIBRIUM (NOT OPTICALLY PUMPED)

GaAs

Theory: **Neilson** (Antwerp), **Perali** (Camerino).
Experiment: **Das Gupta**, **Croxall** (Cambridge), **Lilly** (Sandia), **Hamilton** (Sydney)

Double bilayer graphene

Theory: **Conti**, **Neilson**, **Perali** (Camerino)
Exp: **Tutuc** (Texas), **Dean** (Columbia)

Double TMD monolayers

Theory: **Fogler** (San Diego), **Conti**, **Neilson** (Antwerp), **Perali** (Camerino).
Experiment: **Mak** (Cornell), **Kim** (Harvard), **Schaibley** (Arizona)

Strained Si/Ge

Theory: **Conti**, **Neilson**
Exp: **Scappucci** (Delft)

Anomalous Coulomb Drag in Electron-Hole Bilayers

A. F. Croxall, K. Das Gupta,* C. A. Nicoll, M. Thangaraj, H. E. Beere, I. Farrer, D. A. Ritchie, and M. Pepper

Cavendish Laboratory, University of Cambridge, J.J. Thomson Avenue, Cambridge CB3 0HE, United Kingdom Selected for a Viewpoint in Physics

PHYSICAL REVIEW LETTERS

week ending
22 JULY 2016PRL **117**, 046802 (2016)**Negative Coulomb Drag in Double Bilayer Graphene**J. I. A. Li,¹ T. Taniguchi,² K. Watanabe,² J. Hone,³ A. Levchenko,⁴ and C. R. Dean¹PRL **102**, 026804 (2009)

PHYSICAL REVIEW LETTERS

week ending
16 JANUARY 2009**Coulomb Drag in the Exciton Regime in Electron-Hole Bilayers**

J. A. Seamons, C. P. Morath, J. L. Reno, and M. P. Lilly*

Sandia National Laboratories, Albuquerque, New Mexico 87185, USA

(Received 8 August 2008; published 14 January 2009)

Strong Coulomb drag and broken symmetry in double-layer grapheneR. V. Gorbachev, A. K. Geim , M. I. Katsnelson, K. S. Novoselov, T. Tudorovskiy, I. V. Grigorieva, A. H. MacDonald, S. V. Morozov, K. Watanabe, T. Taniguchi & L. A. Ponomarenko *Nature Physics* **8**, 896–901 (2012) | [Cite this article](#)PHYSICAL REVIEW LETTERS **120**, 177702 (2018)

Editors' Suggestion

Featured in Physics

Strongly Enhanced Tunneling at Total Charge Neutrality in Double-Bilayer Graphene-WSe₂ Heterostructuresam Burg,¹ Nitin Prasad,¹ Kyoungwan Kim,¹ Takashi Taniguchi,² Kenji Watanabe,² Allan H. MacDonald,³ Leonard F. Register,¹ and Emanuel Tutuc^{1,*}**Evidence of high-temperature exciton condensation in two-dimensional atomic double layers**Zefang Wang, Daniel A. Rhodes, Kenji Watanabe, Takashi Taniguchi, James C. Hone, Jie Shan  & Kin Fai Mak *Nature* **574**, 76–80 (2019) | [Cite this article](#)

Anomalous Coulomb Drag inA. F. Croxall, K. Das Gupta,* C. A. Nicoll, M. Thangaraj, I.
Cavendish Laboratory, University of Cambridge, J.J. Thomson Avenue, Cambridge CB3 0FF, United Kingdom

[Submitted on 26 Sep 2023]

Perfect Coulomb drag in a dipolar excitonic insulator

Phuong X. Nguyen, Liguang Ma, Raghav Chaturvedi, Kenji Watanabe, Takashi Taniguchi, Jie Shan, Kin Fai Mak

July

SCIENCE ADVANCES | RESEARCH ARTICLE

Sci. Adv. **11**, eadr1772 (2025) 3 January 2025**Imaging interlayer exciton superfluidity in a 2D semiconductor heterostructure**Jacob Cutshall^{1†}, Fateme Mahdikhany^{1,2†}, Anna Roche¹, Daniel N. Shanks¹, Michael R. Koehler³, David G. Mandrus^{4,5,6}, Takashi Taniguchi⁷, Kenji Watanabe⁸, Qizhong Zhu^{9,10}, Brian J. LeRoy¹, John R. Schaibley^{1*}**Gly Enhanced Tunneling at Total Charge Neutrality in Double-Bilayer Graphene-WSe₂ Heterostructures**William Burg,¹ Nitin Prasad,¹ Kyoungwan Kim,¹ Takashi Taniguchi,² Kenji Watanabe,² Allan H. MacDonald,³ Leonard F. Register,¹ and Emanuel Tutuc^{1,*}

J. A. Seamons, C. P. Morath, J. L. Reno, and M. P. Lilly*

Sandia National Laboratories, Albuquerque, New Mexico 87185, USA

Ultraviolet interlayer excitons in bilayer WSe₂Kai-Qiang Lin , Paulo E. Faria Junior, Ruven Hübner, Jonas D. Ziegler, Jonas M. Bauer, Fa
Matthias Florian, Felix Hofmann, Kenji Watanabe, Takashi Taniguchi, Jaroslav Fabian, Alex
Alexey Chernikov, Sebastian Bange & John M. Lupton*Nature Nanotechnology* **19**, 196–201 (2024) | [Cite this article](#)6028 Accesses | 1 Citations | 13 Altmetric | [Metrics](#)**Evidence of high-temperature exciton condensation in two-dimensional atomic double layers**Zefang Wang, Daniel A. Rhodes, Kenji Watanabe, Takashi Taniguchi, James C.
Mak *Nature* **574**, 76–80 (2019) | [Cite this article](#)**Excitons stabilize above the band gap in bilayer WSe₂**Nihit Saigal & Ursula Wurstbauer *Nature Nanotechnology* **19**, 141–142 (2024) | [Cite this article](#)1989 Accesses | 13 Altmetric | [Metrics](#)



Layer 2: 2×2 Conv

Yin-Chengyu¹, Huiyi Li², Yanyan Jiao³, Xuesi Huo⁴, Jieqi Li⁵, Jingyi Li⁶, Jieqi Li⁷, Xueyi Li⁸, Yanyan Jiao⁹, Huiyi Li¹⁰, Yanyan Jiao¹¹, Xuesi Huo¹², Jieqi Li¹³, Jingyi Li¹⁴, Jieqi Li¹⁵, Xueyi Li¹⁶, Yanyan Jiao¹⁷, Huiyi Li¹⁸, Yanyan Jiao¹⁹, Xuesi Huo²⁰, Jieqi Li²¹, Jingyi Li²², Jieqi Li²³, Xueyi Li²⁴, Yanyan Jiao²⁵, Huiyi Li²⁶, Yanyan Jiao²⁷, Xuesi Huo²⁸, Jieqi Li²⁹, Jingyi Li³⁰, Jieqi Li³¹, Xueyi Li³², Yanyan Jiao³³, Huiyi Li³⁴, Yanyan Jiao³⁵, Xuesi Huo³⁶, Jieqi Li³⁷, Jingyi Li³⁸, Jieqi Li³⁹, Xueyi Li⁴⁰, Yanyan Jiao⁴¹, Huiyi Li⁴², Yanyan Jiao⁴³, Xuesi Huo⁴⁴, Jieqi Li⁴⁵, Jingyi Li⁴⁶, Jieqi Li⁴⁷, Xueyi Li⁴⁸, Yanyan Jiao⁴⁹, Huiyi Li⁵⁰, Yanyan Jiao⁵¹, Xuesi Huo⁵², Jieqi Li⁵³, Jingyi Li⁵⁴, Jieqi Li⁵⁵, Xueyi Li⁵⁶, Yanyan Jiao⁵⁷, Huiyi Li⁵⁸, Yanyan Jiao⁵⁹, Xuesi Huo⁶⁰, Jieqi Li⁶¹, Jingyi Li⁶², Jieqi Li⁶³, Xueyi Li⁶⁴, Yanyan Jiao⁶⁵, Huiyi Li⁶⁶, Yanyan Jiao⁶⁷, Xuesi Huo⁶⁸, Jieqi Li⁶⁹, Jingyi Li⁷⁰, Jieqi Li⁷¹, Xueyi Li⁷², Yanyan Jiao⁷³, Huiyi Li⁷⁴, Yanyan Jiao⁷⁵, Xuesi Huo⁷⁶, Jieqi Li⁷⁷, Jingyi Li⁷⁸, Jieqi Li⁷⁹, Xueyi Li⁸⁰, Yanyan Jiao⁸¹, Huiyi Li⁸², Yanyan Jiao⁸³, Xuesi Huo⁸⁴, Jieqi Li⁸⁵, Jingyi Li⁸⁶, Jieqi Li⁸⁷, Xueyi Li⁸⁸, Yanyan Jiao⁸⁹, Huiyi Li⁹⁰, Yanyan Jiao⁹¹, Xuesi Huo⁹², Jieqi Li⁹³, Jingyi Li⁹⁴, Jieqi Li⁹⁵, Xueyi Li⁹⁶, Yanyan Jiao⁹⁷, Huiyi Li⁹⁸, Yanyan Jiao⁹⁹, Xuesi Huo¹⁰⁰, Jieqi Li¹⁰¹, Jingyi Li¹⁰², Jieqi Li¹⁰³, Xueyi Li¹⁰⁴, Yanyan Jiao¹⁰⁵, Huiyi Li¹⁰⁶, Yanyan Jiao¹⁰⁷, Xuesi Huo¹⁰⁸, Jieqi Li¹⁰⁹, Jingyi Li¹¹⁰, Jieqi Li¹¹¹, Xueyi Li¹¹², Yanyan Jiao¹¹³, Huiyi Li¹¹⁴, Yanyan Jiao¹¹⁵, Xuesi Huo¹¹⁶, Jieqi Li¹¹⁷, Jingyi Li¹¹⁸, Jieqi Li¹¹⁹, Xueyi Li¹²⁰, Yanyan Jiao¹²¹, Huiyi Li¹²², Yanyan Jiao¹²³, Xuesi Huo¹²⁴, Jieqi Li¹²⁵, Jingyi Li¹²⁶, Jieqi Li¹²⁷, Xueyi Li¹²⁸, Yanyan Jiao¹²⁹, Huiyi Li¹³⁰, Yanyan Jiao¹³¹, Xuesi Huo¹³², Jieqi Li¹³³, Jingyi Li¹³⁴, Jieqi Li¹³⁵, Xueyi Li¹³⁶, Yanyan Jiao¹³⁷, Huiyi Li¹³⁸, Yanyan Jiao¹³⁹, Xuesi Huo¹⁴⁰, Jieqi Li¹⁴¹, Jingyi Li¹⁴², Jieqi Li¹⁴³, Xueyi Li¹⁴⁴, Yanyan Jiao¹⁴⁵, Huiyi Li¹⁴⁶, Yanyan Jiao¹⁴⁷, Xuesi Huo¹⁴⁸, Jieqi Li¹⁴⁹, Jingyi Li¹⁵⁰, Jieqi Li¹⁵¹, Xueyi Li¹⁵², Yanyan Jiao¹⁵³, Huiyi Li¹⁵⁴, Yanyan Jiao¹⁵⁵, Xuesi Huo¹⁵⁶, Jieqi Li¹⁵⁷, Jingyi Li¹⁵⁸, Jieqi Li¹⁵⁹, Xueyi Li¹⁶⁰, Yanyan Jiao¹⁶¹, Huiyi Li¹⁶², Yanyan Jiao¹⁶³, Xuesi Huo¹⁶⁴, Jieqi Li¹⁶⁵, Jingyi Li¹⁶⁶, Jieqi Li¹⁶⁷, Xueyi Li¹⁶⁸, Yanyan Jiao¹⁶⁹, Huiyi Li¹⁷⁰, Yanyan Jiao¹⁷¹, Xuesi Huo¹⁷², Jieqi Li¹⁷³, Jingyi Li¹⁷⁴, Jieqi Li¹⁷⁵, Xueyi Li¹⁷⁶, Yanyan Jiao¹⁷⁷, Huiyi Li¹⁷⁸, Yanyan Jiao¹⁷⁹, Xuesi Huo¹⁸⁰, Jieqi Li¹⁸¹, Jingyi Li¹⁸², Jieqi Li¹⁸³, Xueyi Li¹⁸⁴, Yanyan Jiao¹⁸⁵, Huiyi Li¹⁸⁶, Yanyan Jiao¹⁸⁷, Xuesi Huo¹⁸⁸, Jieqi Li¹⁸⁹, Jingyi Li¹⁹⁰, Jieqi Li¹⁹¹, Xueyi Li¹⁹², Yanyan Jiao¹⁹³, Huiyi Li¹⁹⁴, Yanyan Jiao¹⁹⁵, Xuesi Huo¹⁹⁶, Jieqi Li¹⁹⁷, Jingyi Li¹⁹⁸, Jieqi Li¹⁹⁹, Xueyi Li²⁰⁰, Yanyan Jiao²⁰¹, Huiyi Li²⁰², Yanyan Jiao²⁰³, Xuesi Huo²⁰⁴, Jieqi Li²⁰⁵, Jingyi Li²⁰⁶, Jieqi Li²⁰⁷, Xueyi Li²⁰⁸, Yanyan Jiao²⁰⁹, Huiyi Li²¹⁰, Yanyan Jiao²¹¹, Xuesi Huo²¹², Jieqi Li²¹³, Jingyi Li²¹⁴, Jieqi Li²¹⁵, Xueyi Li²¹⁶, Yanyan Jiao²¹⁷, Huiyi Li²¹⁸, Yanyan Jiao²¹⁹, Xuesi Huo²²⁰, Jieqi Li²²¹, Jingyi Li²²², Jieqi Li²²³, Xueyi Li²²⁴, Yanyan Jiao²²⁵, Huiyi Li²²⁶, Yanyan Jiao²²⁷, Xuesi Huo²²⁸, Jieqi Li²²⁹, Jingyi Li²³⁰, Jieqi Li²³¹, Xueyi Li²³², Yanyan Jiao²³³, Huiyi Li²³⁴, Yanyan Jiao²³⁵, Xuesi Huo²³⁶, Jieqi Li²³⁷, Jingyi Li²³⁸, Jieqi Li²³⁹, Xueyi Li²⁴⁰, Yanyan Jiao²⁴¹, Huiyi Li²⁴², Yanyan Jiao²⁴³, Xuesi Huo²⁴⁴, Jieqi Li²⁴⁵, Jingyi Li²⁴⁶, Jieqi Li²⁴⁷, Xueyi Li²⁴⁸, Yanyan Jiao²⁴⁹, Huiyi Li²⁵⁰, Yanyan Jiao²⁵¹, Xuesi Huo²⁵², Jieqi Li²⁵³, Jingyi Li²⁵⁴, Jieqi Li²⁵⁵, Xueyi Li²⁵⁶, Yanyan Jiao²⁵⁷, Huiyi Li²⁵⁸, Yanyan Jiao²⁵⁹, Xuesi Huo²⁶⁰, Jieqi Li²⁶¹, Jingyi Li²⁶², Jieqi Li²⁶³, Xueyi Li²⁶⁴, Yanyan Jiao²⁶⁵, Huiyi Li²⁶⁶, Yanyan Jiao²⁶⁷, Xuesi Huo²⁶⁸, Jieqi Li²⁶⁹, Jingyi Li²⁷⁰, Jieqi Li²⁷¹, Xueyi Li²⁷², Yanyan Jiao²⁷³, Huiyi Li²⁷⁴, Yanyan Jiao²⁷⁵, Xuesi Huo²⁷⁶, Jieqi Li²⁷⁷, Jingyi Li²⁷⁸, Jieqi Li²⁷⁹, Xueyi Li²⁸⁰, Yanyan Jiao²⁸¹, Huiyi Li²⁸², Yanyan Jiao²⁸³, Xuesi Huo²⁸⁴, Jieqi Li²⁸⁵, Jingyi Li²⁸⁶, Jieqi Li²⁸⁷, Xueyi Li²⁸⁸, Yanyan Jiao²⁸⁹, Huiyi Li²⁹⁰, Yanyan Jiao²⁹¹, Xuesi Huo²⁹², Jieqi Li²⁹³, Jingyi Li²⁹⁴, Jieqi Li²⁹⁵, Xueyi Li²⁹⁶, Yanyan Jiao²⁹⁷, Huiyi Li²⁹⁸, Yanyan Jiao²⁹⁹, Xuesi Huo³⁰⁰, Jieqi Li³⁰¹, Jingyi Li³⁰², Jieqi Li³⁰³, Xueyi Li³⁰⁴, Yanyan Jiao³⁰⁵, Huiyi Li³⁰⁶, Yanyan Jiao³⁰⁷, Xuesi Huo³⁰⁸, Jieqi Li³⁰⁹, Jingyi Li³¹⁰, Jieqi Li³¹¹, Xueyi Li³¹², Yanyan Jiao³¹³, Huiyi Li³¹⁴, Yanyan Jiao³¹⁵, Xuesi Huo³¹⁶, Jieqi Li³¹⁷,

1000

Player WSe₂

Superfluid equations

$$V_{\mathbf{q}}^S = V_{\mathbf{q}}^{ee} = V_{\mathbf{q}}^{hh} = \frac{2\pi e^2}{4\pi\epsilon\epsilon_0|\mathbf{q}|} \text{ and } V_{\mathbf{q}}^D = V_{\mathbf{q}}^{eh} = V_{\mathbf{q}}^{he} = -V_{\mathbf{q}}^S e^{-|\mathbf{q}|d}$$

$$\Delta_{\mathbf{k}} = -\frac{1}{A} \sum_{\mathbf{k}'} V_{\mathbf{k}-\mathbf{k}'}^D \frac{\Delta_{\mathbf{k}'}}{2E_{\mathbf{k}'}}$$
$$n = \frac{g}{A} \sum_{\mathbf{k}} v_{\mathbf{k}}^2.$$

BCS mean-field is a good approximation at $T=0$ for strong- and weak-coupling superfluidity

P. Nozières & F. Pistolesi, *From semiconductors to superconductors*, Euro. Phys. B **10** 649 (1999)

energies $E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2}$, $\xi_{\mathbf{k}} = \varepsilon_{\mathbf{k}} - \mu$

Bogoliubov factors, $v_k = \sqrt{\frac{1}{2}(1 - \xi_k/E_k)}$ $u_k = \sqrt{\frac{1}{2}(1 + \xi_k/E_k)}$

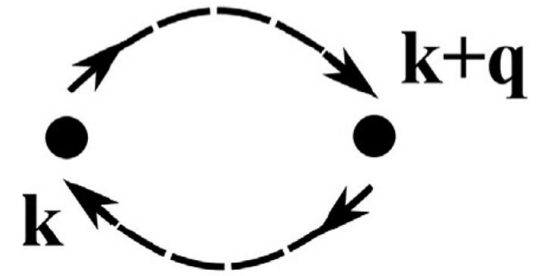
Screening in the superfluid

Screening of the long-range Coulomb attraction for the pairs is a major challenge

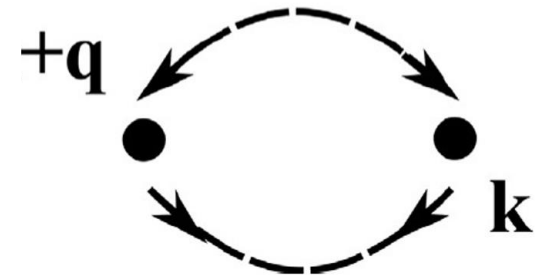
$$G(\mathbf{k}, i\varepsilon_n) = \frac{u_k^2}{i\varepsilon_n - E_k} + \frac{v_k^2}{i\varepsilon_n + E_k}$$

$$F(\mathbf{k}, i\varepsilon_n) = \frac{u_k v_k}{i\varepsilon_n - E_k} - \frac{u_k v_k}{i\varepsilon_n + E_k}$$

$$\begin{aligned} \Pi_0^N(\mathbf{q}) &= \frac{ig}{\hbar A} \sum_{\mathbf{k}} \lim_{\eta, \omega \rightarrow 0} \int d\tau e^{i(\omega\tau + i\eta|\tau|)} iG_{\mathbf{k}+\mathbf{q}}(\tau) iG_{\mathbf{k}}(-\tau) \\ &= -\frac{g}{A} \sum_{\mathbf{k}} \frac{u_{\mathbf{k}}^2 v_{\mathbf{k}+\mathbf{q}}^2 + v_{\mathbf{k}}^2 u_{\mathbf{k}+\mathbf{q}}^2}{E_{\mathbf{k}} + E_{\mathbf{k}+\mathbf{q}}}, \end{aligned}$$



$$\begin{aligned} \Pi_0^A(\mathbf{q}) &= -\frac{ig}{\hbar A} \sum_{\mathbf{k}} \lim_{\eta, \omega \rightarrow 0} \int d\tau e^{i(\omega\tau + i\eta|\tau|)} iF_{\mathbf{k}+\mathbf{q}}^*(\tau) iF_{\mathbf{k}}(-\tau) \\ &= -\frac{2g}{A} \sum_{\mathbf{k}} \frac{u_{\mathbf{k}} v_{\mathbf{k}} v_{\mathbf{k}+\mathbf{q}} u_{\mathbf{k}+\mathbf{q}}}{E_{\mathbf{k}} + E_{\mathbf{k}+\mathbf{q}}}. \end{aligned}$$



Screening in the superfluid

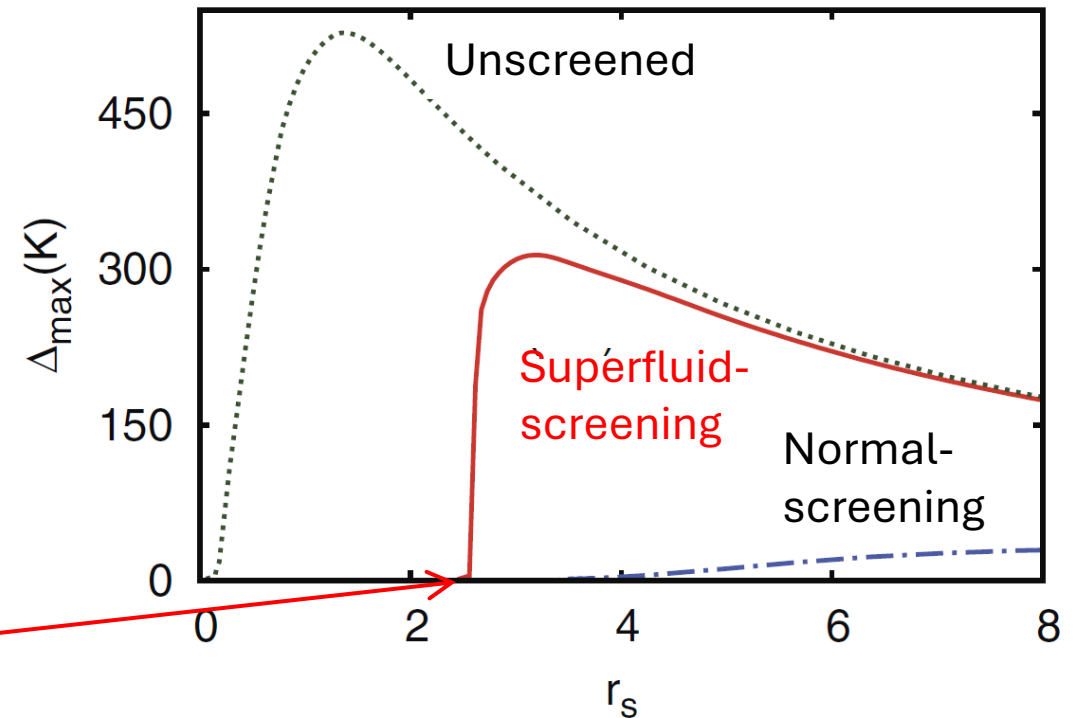
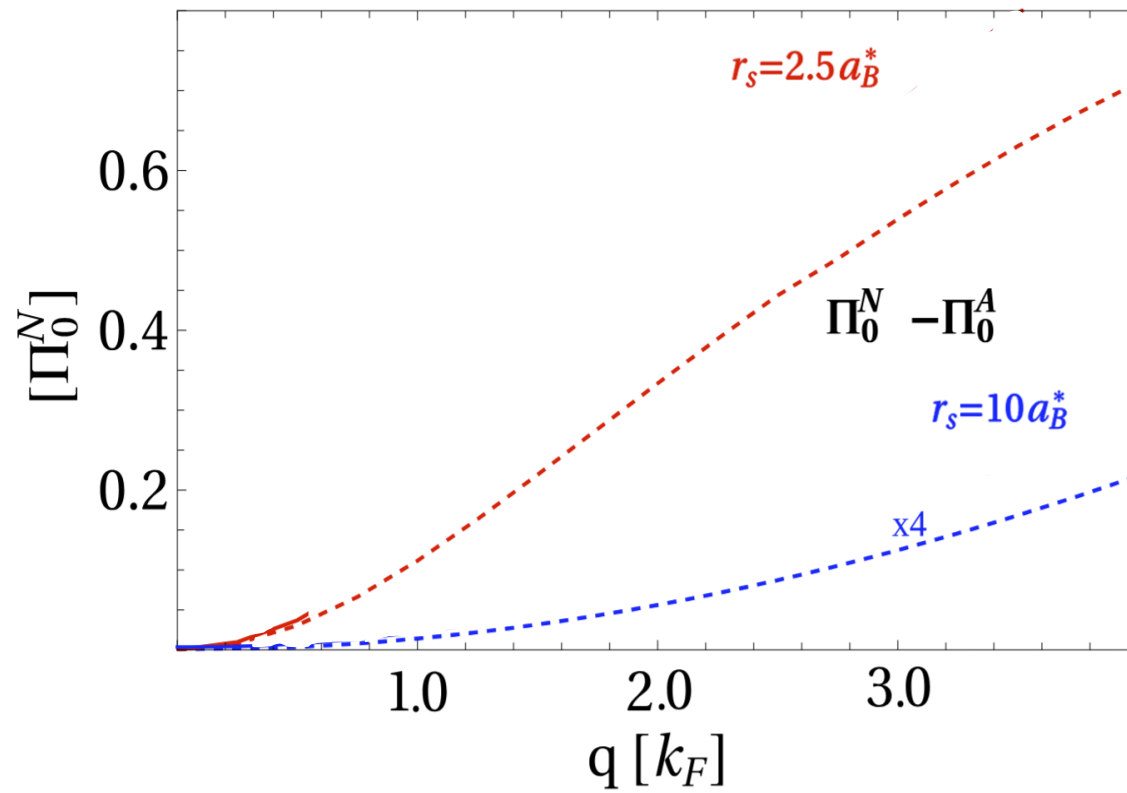
$$\text{wavy line} = \text{bare wavy line} + \text{loop with shaded square}$$

$$\text{wavy line} = \text{bare wavy line} + \text{loop with 2 squares} + \text{loop with 3 squares} + \dots$$

$$V_{\mathbf{q}}^{S,RPA} = \frac{V_{\mathbf{q}}^S - \Pi_0^N(\mathbf{q})\mathcal{A}_{\mathbf{q}}}{1 - 2(\Pi_0^N(\mathbf{q})V_{\mathbf{q}}^S + \Pi_0^A(\mathbf{q})V_{\mathbf{q}}^D) + \mathcal{A}_{\mathbf{q}}((\Pi_0^N(\mathbf{q}))^2 - (\Pi_0^A(\mathbf{q}))^2)}$$

$$V_{\mathbf{q}}^{D,RPA} = \frac{V_{\mathbf{q}}^D + \Pi_0^A(\mathbf{q})\mathcal{A}_{\mathbf{q}}}{1 - 2(\Pi_0^N(\mathbf{q})V_{\mathbf{q}}^S + \Pi_0^A(\mathbf{q})V_{\mathbf{q}}^D) + \mathcal{A}_{\mathbf{q}}((\Pi_0^N(\mathbf{q}))^2 - (\Pi_0^A(\mathbf{q}))^2)}$$

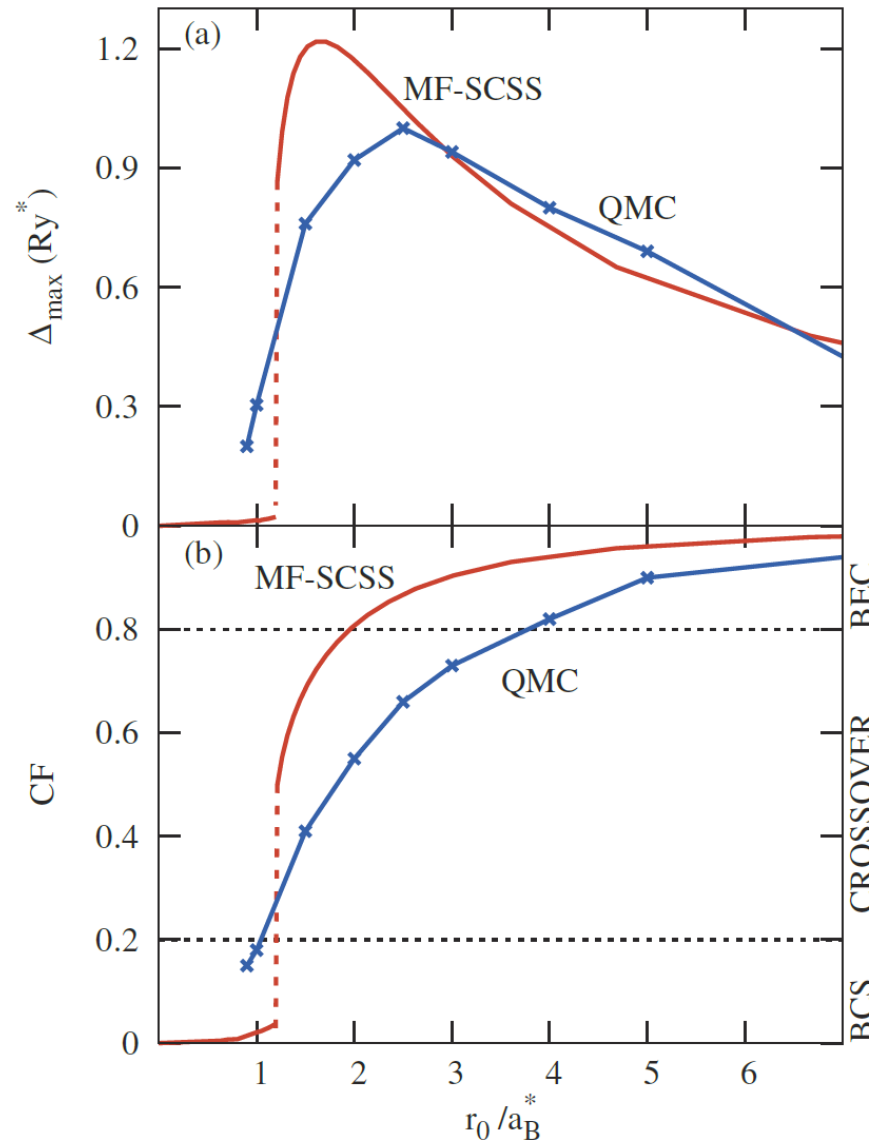
with $\mathcal{A}_{\mathbf{q}} = (V_{\mathbf{q}}^S)^2 - (V_{\mathbf{q}}^D)^2$ and $\mathcal{B}_{\mathbf{q}} \quad V_{\mathbf{q}}^D = -V_{\mathbf{q}}^S e^{-|\mathbf{q}|d}$



Electron-hole superfluidity is a low-density phenomenon

Gap, onset density, and crossover physics

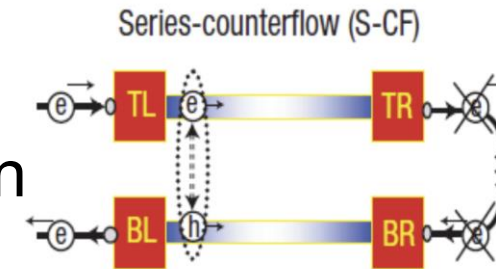
Across superfluid region, the
Condensate Fraction $CF \gtrsim 0.2$
 $\Rightarrow$ no BCS regime



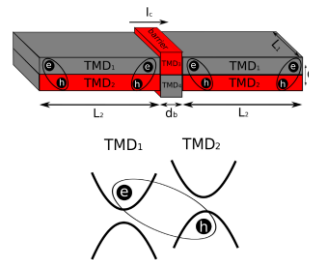
There has been no definite experimental claim of superfluidity

It is *timely* to find *fingerprints* to *unambiguously* identify the superfluidity

- ~~○ Steady state dissipationless counterflow current cannot be straightforwardly induced because of increasing interlayer tunnelling and e-h recombination~~
- ~~○ No Meissner effect in a neutral system~~

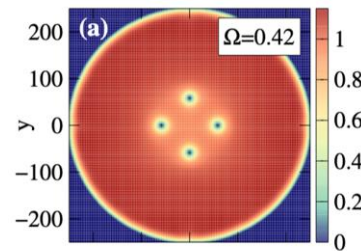


Could use:



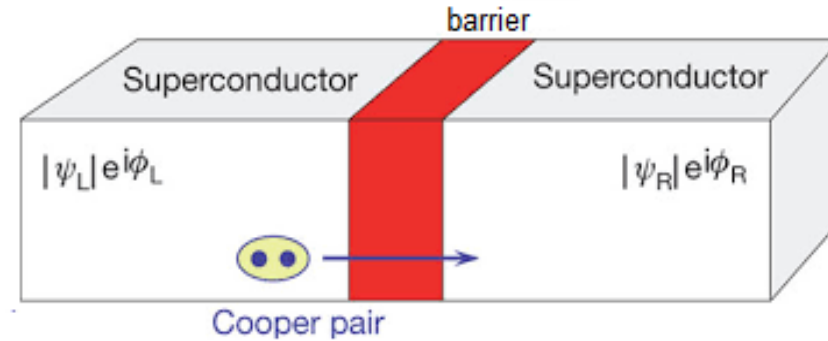
✓ Josephson ✓

✓ Detection of vortices ✓

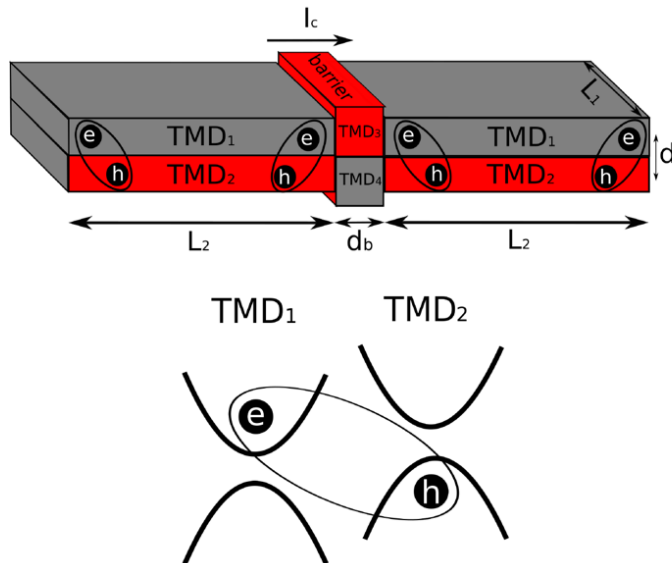


✓ Additional collective modes ✓

Josephson effect



Dissipationless macroscopic coherent current of Cooper pairs without a driving potential flowing between two superconductors separated by a barrier.



DC JE: Measurement of a super tunneling current and zero voltage drop across the barrier.

Problems: Difficult to measure very low values of resistance and voltage.

Possible solution: **Shapiro steps.**

AC JE: Highly oscillating super current in the barrier region related to the time evolution of the phase order parameter.

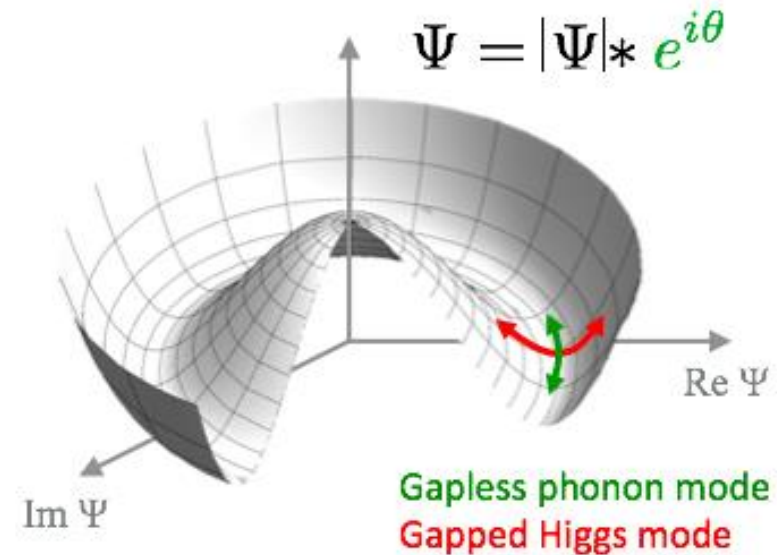
Collective modes

From small perturbations around the ground state - of what?.

Order parameter:

$$\Psi_0 = |\Psi_0|e^{i\theta_0}$$

- Phase fluctuations $\theta = \theta_0 + \delta\theta$:
Gapless phonon mode.
- Amplitude fluctuations $|\Psi| = |\Psi_0| + |\delta\Psi|$:
Gapped pair-breaking mode.



Phase and Amplitude collective modes **only** in the superfluid phase.

Density collective modes from normal to superfluid phase.

Vortices

In the BEC regime, exciton-exciton repulsive interaction is unscreened:

$$V_{XX}(r) = \frac{2e^2}{4\pi\epsilon} \left(\frac{1}{r} - \frac{1}{\sqrt{r^2 + d^2}} \right)$$

Vortex properties and numbers will depend on interaction strength $\sim d^2$

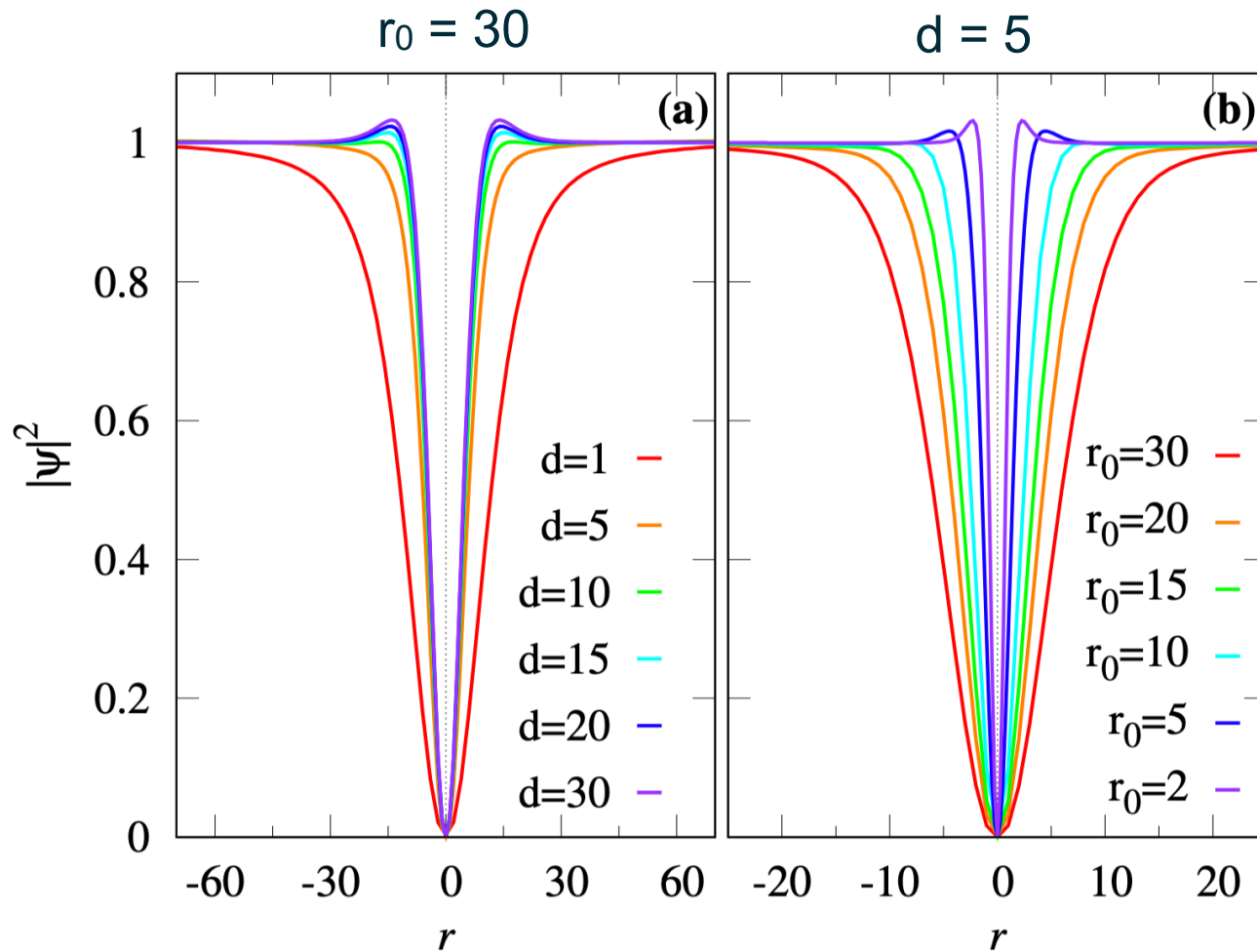
→ Gross-Pitaevskii equation for superfluid in rotating frame

$$-\frac{\hbar^2}{2M} \nabla^2 \Psi(\mathbf{r}) + \phi_{XX}(\mathbf{r}) \Psi(\mathbf{r}) - \mathbf{\Omega} \cdot \mathbf{L} \Psi(\mathbf{r}) = \mu \Psi(\mathbf{r})$$

$$\phi_{XX}(\mathbf{r}) = \int V_{XX}(|\mathbf{r} - \mathbf{r}'|) |\Psi(\mathbf{r}')|^2 d\mathbf{r}'$$

$\mathbf{\Omega}$: angular velocity

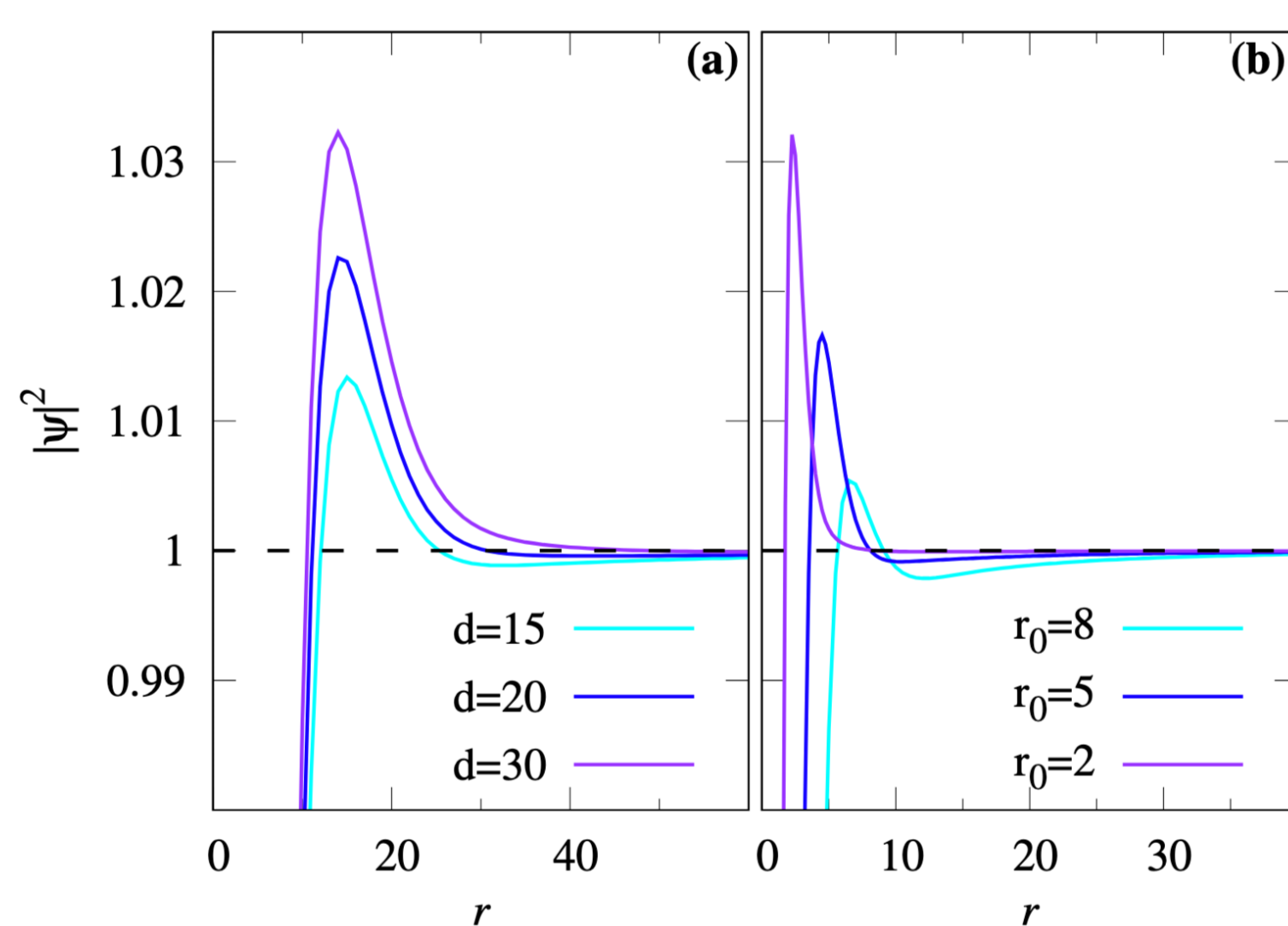
Vortices



With increasing dipole strength d or increasing density:

- Vortex radius decreases
- A density pileup peak appears at vortex edge

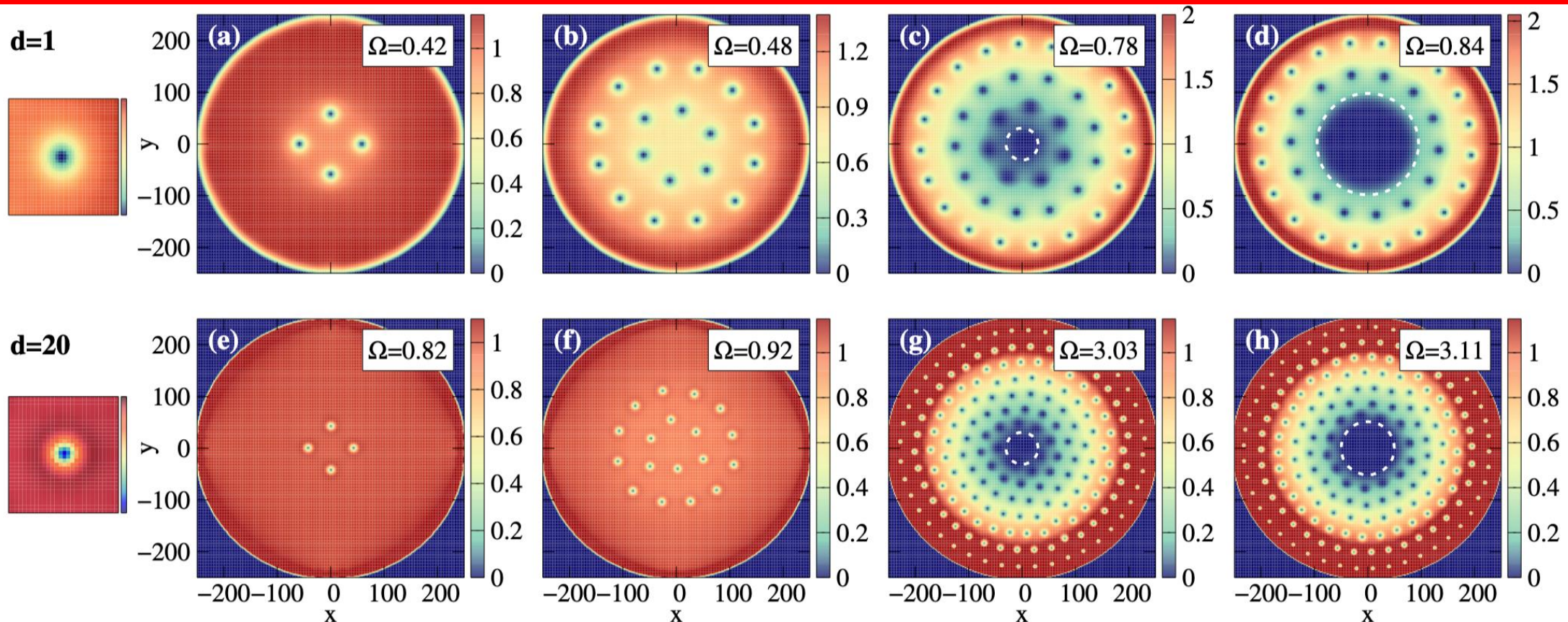
Vortices



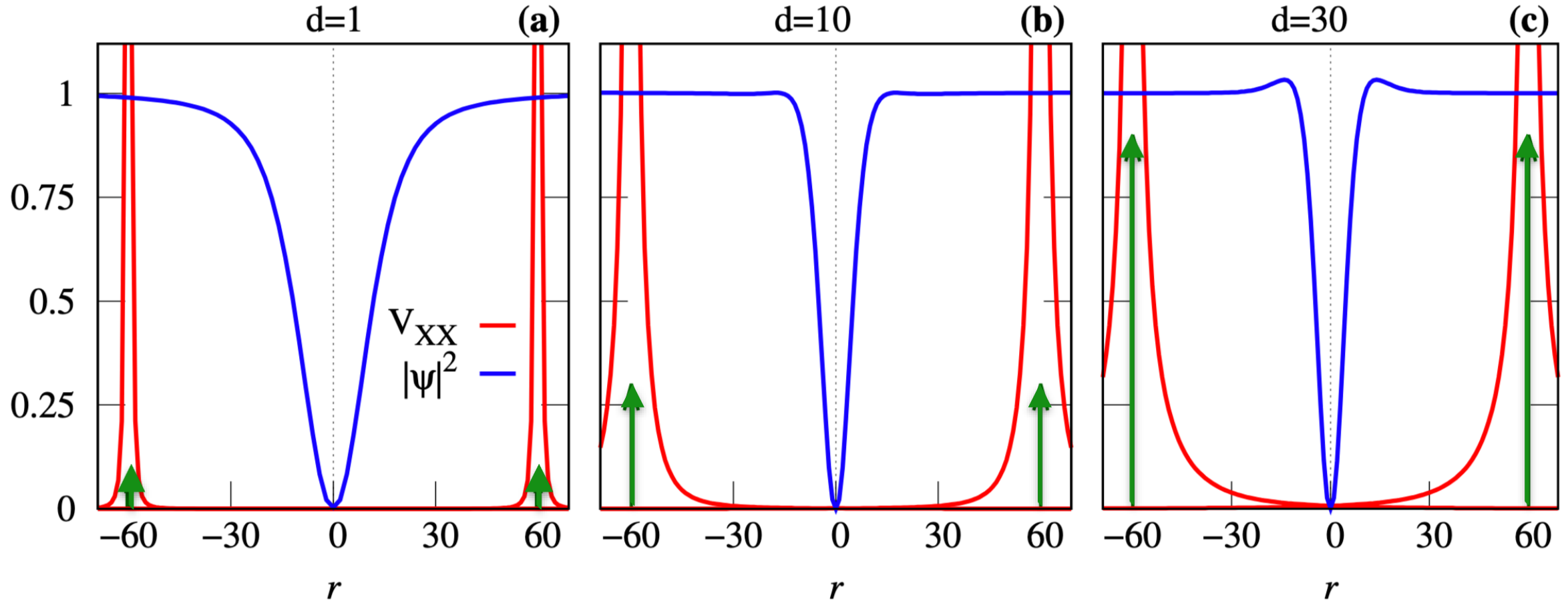
- Cold atoms with dipolar interactions also display a pileup peak when density increases.
- But in cold atoms the peak is accompanied by oscillations further out – associated with a low-lying roton collective excitation.
- **No oscillations** → suggests: **no roton** for pure repulsion

Vortex Lattices

- The vortices for $d=20$ are smaller than for $d = 1$
- The threshold Ω when the vortices overlap is larger for $d=20$ than $d = 1$
- The vortices for $d = 20$ have wings

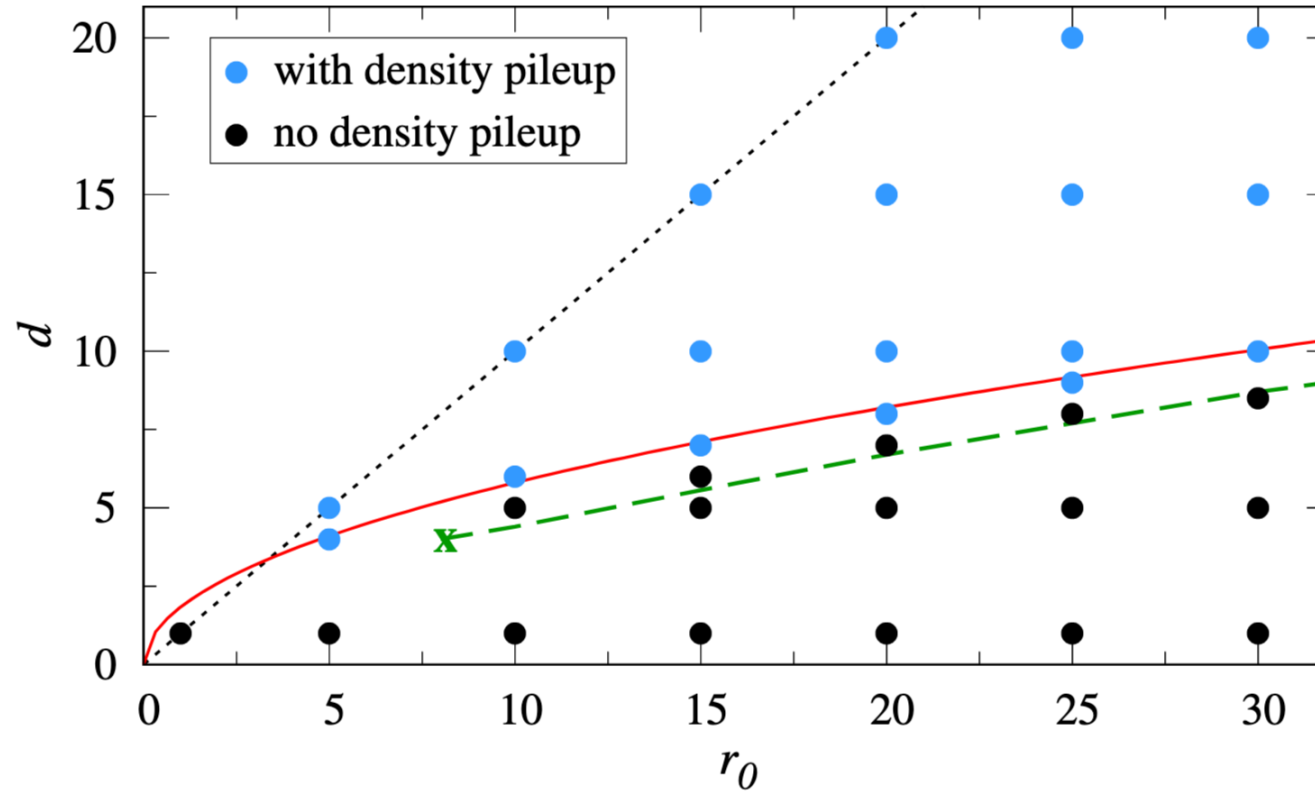


Vortices



The accumulation of density at the vortex edge comes from the competition between the **outward centrifugal force** and the **inward repulsion from neighbouring dipoles**.

Vortices



RED LINE: density pileup appears on vortex.

GREEN LINE: Gross Pitaevskii transition from superfluid to supersolid

Both density pileup and transition occur as the effective range of the interaction becomes of the same order as r_0 .